Essays on Corporate Finance and Investment

A thesis submitted for the degree
of Doctor of Philosophy of
the Australian National University

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February 28, 2002

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university. To the best of the author's knowledge and belief, it contains no material previously published or written by another person, except where due reference is made in the text. Chapters 4, 5 and 6 of the thesis resulted from joint work with fellow student, Andrew Wait.

Vladimir Smirnov

February 28, 2002

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Acknowledgements

The results presented in this thesis were obtained in my three-years of research conducted at the Australian National University. It is hard to express the extent of my gratitude to my supervisor, Dr. Rohan Pitchford. From the very start and up to now, I have always felt his wise guidance, understanding and readiness to help. Thank you very much, my dear boss! I would also like to thank my advisors, Professor Steve Dowrick and Professor Simon Grant.

Other members of the academic staff and visitors to the ANU had enormous influence on my progress. Among them I would particularly like to thank Matt Benge, Bob Gregory, Chris Jones, Flavio Menezes, Martin Osbourne, Matthew Ryan, Ben Smith, Rhema Vaithianathan and participants in RSSS/PhD seminars and Economic Theory workshops.

I would also like to acknowledge my debt to classmates, colleges and friends at ANU. I would especially like to thank Andrew Wait, not only as my co-author, but also for his tireless efforts reading this document. Thanks also goes to Rhema Vaithianathan, Kris Funston and Natalia Ponomareva. Their efforts are greatly appreciated by me.

Finally, my largest debt of gratitude is to my family for their patience and love. Without them this work would never have come into existence.

Abstract

This thesis makes three important theoretical contributions to the existing literature. First, the thesis explores the hold-up problem between two parties (an entrepreneur and an investor) when one of the parties (the entrepreneur) is unable to commit not to repudiate the initial contract. As in Neher (1999), we allow the parties to stage investments over time to help mitigate the hold-up problem. However, unlike Neher, we derive the optimal investment path for a variable rate of return. We show that when the rate of return is not constant the optimal investment path is significantly different. For example, our model predicts that neither positive wealth of the entrepreneur nor the lack of discounting ensures that all profitable projects proceed. The model is extended in several ways: first, both agents are allowed to repudiate the initial contract; and, second, new costs of staged financing are introduced.

Second, the thesis explores the hold-up problem when trading parties can choose to make specific investments simultaneously or sequentially. An advantage of staging investments is contracting on any subsequent investment becomes possible after the project is underway and better defined. It is shown that there can be efficiency improvements with the sequential regime, as compared with simultaneous investment, if the parties are sufficiently patient. Further, as previously emphasized in the literature, sequencing of investments can allow some projects to proceed that would not be feasible with a simultaneous regime. This however, is not always the case. A cost of sequencing investment is that it can disadvantage some parties, reducing their incentive to invest. In fact, the mere possibility of sequential investment can

be detrimental to overall welfare. In the extreme it can prevent mutually beneficial trade from occurring. This is a new result: it allows the choice about the timing of investment to be interpreted as a new (potential) form of hold-up. Additionally, in a continuous set-up when the two investments are independent, three effects are identified when comparing the two regimes: sequential investment increases the costs of delay; sequential investment reduces the incentive for the first player to invest; and the sequential regime increases the second player's incentive to invest.

Third, the thesis also examines a dynamic investment game where industry sunk costs provide an incentive for a firm to be a follower into the market as opposed to a leader. Interesting dynamics can arise: as the potential investment horizon is extended the game can switch from a prisoners' dilemma to a coordination game and back again. The model also exhibits an investment cascade: once one firm has entered the market all other firms enter immediately after. This result arises without the presence of asymmetric information.

CHAPTER 1

Introduction

Investment is an essential economic activity. It creates economic growth and generates surplus. However, investment is often undertaken when complete contracts between invested parties are not possible. As a result agents are unable to commit to their future behavior. Such incompleteness can affect the returns to the investor and, consequently, the incentives to invest. Thus, the main goal of this thesis is to explore the incentive to invest when contracts are incomplete.

In Chapter 2 we present a selective review of the literature that investigates how a firm chooses the debt-to-equity ratio when it needs to finance investment projects. This provides an insight into the optimal amount of debt that a firm will choose when undertaking any investment. We divide the material of this chapter into five sections. First, we discuss models in which capital structure is determined by agency costs. Then, we consider how the presence of incomplete information influences the optimal debt level. Further, we describe models in which corporate structure acts as a strategic tool. In these models firms use debt to either gain advantage over rivals or to commit to future behavior. Additionally, we outline models that focus on corporate control considerations. In the final section, we review the literature that provides insight into the optimal structure of debt using incomplete contract theory approach.

Chapter 3 presents a new contribution to this literature. We explore the hold-up problem between two parties (an entrepreneur and an investor) when one of the parties (the entrepreneur) is unable to commit not to repudiate the initial contract. As in Neher (1999), we allow the parties to stage investments over time to help mitigate the hold-up problem. However, unlike Neher, we derive the optimal investment path for a variable rate of return case. A non-constant rate of return allows for an examination of a wider scope of projects including, for example, projects with fixed costs or increasing and decreasing rates of return. It will be shown that relaxing the assumption of constant returns significantly alters the optimal investment path - this is the main result of the chapter. Although the algorithm of the optimal investment path derivation is quite similar to the algorithm presented in Neher(1999), some of the results presented here are considerably different. For instance, Neher (1999) predicts that positive wealth of the entrepreneur and a lack of discounting (equivalent to a very short time duration of one period) ensures that all profitable projects go forward. On the contrary, the model presented here shows that this is not the case. Specifically, with a variable rate of return, not all profitable projects are financed, even if the parties can stage their investments.

In this chapter we also explore the issue of repudiation in more detail. We construct a model in which both agents are able to repudiate, and find that allowing the investor to repudiate mitigates the commitment problem to such an extent that all profitable two-period projects are financed. There is still some inefficiency in this set-up with respect to financing in one period, but this inefficiency is much smaller than in the case when only the entrepreneur has the right to repudiate.

Whereas previous literature has considered the costs of staging simply as the delays of project completion, we introduce additional staging costs such as costs of additional shareholder meetings. We assume, for example, that half of the capital is invested straight away, and the remaining half of the capital is invested in half a period's time. The project finishes in one period, and the sole costs of staging are due to 'chopping' this period in two parts. Surprisingly, we find that the optimal investment path for this model exactly coincides with the optimal investment path for the main model with no discounting. We find that with respect to a model without any costs of staging, adding additional costs of 'chopping' the period, only diminishes the likelihood that the project will be financed. It does not though change the optimal investment path.

The model also generalizes several of Neher's results for projects with a variable rate of return. For example, an increase in the profitability of a project both increases the probability that the project will be financed and weakly decreases the number of stages needed. We derive similar results with respect to a decrease in the discount rate, an increase in tangibility of the physical assets of the project, and an increase in growth of physical assets in outside value. Further, we relax the assumption that the entrepreneur has no wealth and find that it is optimal for him to invest all his wealth at the beginning of the project, before relying on outside financing in subsequent rounds.

Chapter 4 explores the hold-up problem between two trading parties investing in a profitable relationship when they can choose to make specific investments either simultaneously or sequentially. An advantage to staging such investments is that contracting on any subsequent investment becomes possible after the project is under way. We show that if the parties are sufficiently patient, sequential investment can lead to efficiency improvements, as compared with simultaneous investment. Further, sequencing of investments can allow some projects to proceed that would not be feasible with a simultaneous regime.[†] This however is not always the case. A cost of sequencing investment is that it can disadvantage the party that makes the initial investment, reducing their incentive to invest. In fact, the mere possibility of sequential investment can be detrimental to overall welfare. In the extreme it can prevent mutually beneficial trade from occurring. This is a new result: it allows the choice about the timing of investment to be interpreted as a new (potential) form of hold-up.

The decision over the timing of investment can be seen as a choice over the completeness of contracts: if parties opt for simultaneous investment they are opting for a more incomplete contract than necessary. As a result, the choice concerning the completeness of contract is endogenous. The advantage of a (more) complete contract with sequential investment is that hold-up of the follower is avoided. The cost of a complete contract is that it diminishes the first party's incentive to invest.

[†]See, for example, Neher (1999) and Admati and Perry (1991).

A party will opt for simultaneous investment - that is, they will opt for an incomplete contract - when their gain from the increase in total surplus outweighs the additional bargaining power they receive from avoiding hold-up.

Finally, interesting dynamics can arise out of this investment game when both parties want to be a follower rather than the leader. If there are just two potential investment periods (and the opportunity to invest disappears after the second period) the parties find themselves in a prisoners' dilemma. If the potential investment horizon is continually extended to three periods, four periods and so on, eventually the benefit from not investing (waiting) will diminish sufficiently so that the players will find themselves in a coordination game. (The players will mix between investing immediately and waiting.) If the horizon is extended further from this point, it is possible that the players will again return to a prisoners' dilemma game. This arises because the payoff in the coordination game (say in period K) alters the expected return from waiting in the game with the longer horizon (say a game of $K+1$ periods). It is possible that the optimal strategies switch between a prisoners' dilemma game and a coordination game as the potential horizon is extended. As far as I am aware, there are no games in the existing literature that exhibit this sort of dynamic switching.

In Chapter 5 we generalize the model presented in Chapter 4 for continuous values of investments. In this set-up, both the timing and size of investment are endogenous. It is shown in the model that if the two investments are independent, three effects are identified when comparing the total welfare of the two regimes: sequential investment increases the costs of delay; sequential investment reduces the incentive for the first player to invest; and the sequential regime increases the second player's incentive to invest. The ultimate impact on total surplus is a combination of these effects. We analyze the circumstances under which each regime is welfare maximizing. Moreover, despite the simplicity of the model, no simple relationship between the welfare effects of the two regimes exists, as there are no restrictions on how the three effects identified above interact. However, given that the simultaneous regime encourages the first player to invest, if this player's contribution is relatively more important than the other player's contribution the simultaneous regime is preferred. In the same way, the sequential regime is preferred when the second investor's contribution is relatively more important, provided both players are sufficiently patient. Similarly, if a party is not responsive to the incentives provided by one timing regime, the regime that maximizes the other party's incentive to invest is preferred. For example, when the first investor is not responsive to the additional incentive provided by the simultaneous regime, the sequential regime generates a higher level of surplus. On the other hand, if the second player is not responsive to the additional incentives to invest provided by the sequential regime, the simultaneous regime is preferred. These predictions are similar in nature to the property-rights predictions of Hart (1995), although the model is somewhat more general in that a player may voluntarily forgo the advantages of sequencing of investment (their property right) in order to encourage the other party to invest more. In this way, the parties can opt for a (more) incomplete contract by choosing to invest simultaneously. The model is also extended in several other ways, for example, by analyzing these trade-offs when the two investments are strategic complements and substitutes.

Chapter 6 considers the choice between being a leader or a follower in an investment game. We assume that there is a number of potential investors in a new profitable market opportunity. Before any firm can exploit this opportunity, a certain amount of resources needs to be expended on either advertising - to inform the public of the new product - or on non-patented research. This cost is borne by firms that initially enter the market, but this expenditure is a public good for all potential entrants in that, once the investment has been sunk, all firms can benefit of this investment if they choose to enter the market. The question then arises for each firm as to when they should enter the market. Early entry allows them to benefit with fewer competitors, but may mean they incur some of the industry set-up costs, whilst delayed entry may allow a firm to avoid the set-up costs, but they forgo some benefits by not participating in the market.

Several interesting results arise out of the model. First, the switching result of

Chapter 4 is generalized to n players. Namely, it is possible that with two potential investment periods the parties find themselves in a prisoners' dilemma; with three potential investment periods they mix between investing immediately and waiting; and with four potential investment periods the players again return to a prisoners' dilemma. Second, the model can display an entry cascade, quite often observed in empirical studies of technology diffusion. Once one player has entered the market, all other potential suppliers enter as soon as possible. Note that this result is generated when all parties have perfect information.

CHAPTER 2

The theory of capital structure: a literature review

2.1 Introduction

The theory of capital structure started with the celebrated paper on the cost of capital, Modigliani and Miller (1958), in which the authors demonstrated that, in the absence of taxes and frictions and with complete markets, the value of the firm is independent of the financial structure. Since then, a huge volume of research has been conducted in this area. This chapter provides a selective review of the capital structure theory. Our goal here is to describe the factors that determine how a firm chooses the debt/equity ratio when it needs to finance investment projects.[†]

To address this issue we divide the material of this chapter into five sections. First, we discuss models in which capital structure is determined by agency costs. A conflict arises in these models because different agents might have different incentives. Specifically, shareholders maximize the value of the firm, while managers maximize they own utility. Managers have full control of the firm and shareholders can only make initial capital structure decisions. Consequently, shareholders choose the value of debt to maximize ex ante firm value given expected manager's behavior.

In the second section, models with incomplete information are considered. In these models managers possess information about a firm that is not directly available to outsiders (current and potential shareholders). Capital structure decisions can

[†]Debt yields a fixed return to the investor, while equity provides the investor a claim on the returns of the project.

be used by managers to signal the firm's profitability to the capital market.

Third, we examine market equilibrium strategic models. In these models corporate structure acts as a strategic tool. Firms use it to either gain an advantage over rivals or to commit to future behavior, for example with customers or suppliers.

Finally, we outline models that focus on corporate control considerations. In these models corporate structure is used to influence which group of investors, be it the current owners or another company, gets control of the firm.

As an extension to this literature we also consider the issue of the optimal structure of debt contract. Models examining this issue typically have two agents with different objectives, as in the models in section 2.2. The first agent (an investor) provides funds for the second agent (an entrepreneur) to undertake a project. The parties know that renegotiation of the initial debt contract can occur because, either contracts are incomplete or not binding. Namely, by threatening to remove his essential human capital the entrepreneur can alter the distribution of surplus in the project. In this situation, debt contracts need to be renegotiation-proof. This literature provides insight into the optimal structure of debt.

Some of the arguments discussed in this chapter are based on other surveys of the theory of capital structure, particularly Harris and Raviv (1991), Hart (1995) and Shleifer and Vishny (1997).

2.2 Agency Cost

Probably the best paper to describe the research in this area is Jensen and Meckling (1976), who identify two types of conflicts within a firm. First, there is a conflict between the firm's owners and managers.[†] This conflict emerges because while managers incur effort costs, they do not receive all the benefits from their profit enhancement activities.[†] This creates incentives for the managers to put

[†]This is well known conflict between ownership and control.

[†]As in the most of the literature, the owners of the firm (equity holders) are assumed to be interested in its total value. Managers, who have the full control of the firm, are assumed to maximize they own utility (benefits minus costs).

a suboptimal amount of effort into managing firm resources and to transfer the firm resources to their own personal benefits, for example managers may consume 'perquisites' or build 'empires'. An increase in the value of debt, ceteris paribus, mitigates the conflict of interest between shareholders and managers outlined above: an increase in debt increases the share of managers in the firm profits and motivates them to work harder. Consequently, debt financing has a positive effect on firm value because it softens the conflict between shareholders and managers.

Second, there is a conflict between debt holders and equity holders. This conflict arises because debt contracts give equity holders an incentive to invest in risky projects. On the one hand, if these investments succeed all of the marginal benefits (after paying the debt holders their fixed return) will be captured by equity holders. On the other hand, if these investments do not succeed some of the costs will be borne by debt holders because equity holders have limited liabilities. Debt holders anticipate this and impose additional costs on equity holders by increasing costs of debt. This effect is called the 'asset substitution effect'. A decrease in the value of debt mitigates the conflict between debt holders and equity holders outlined above. It decreases the value that can be extracted from debt holders in the case of bankruptcy, makes it unprofitable for shareholders to finance very risky projects and consequently, leads to a reduction in the additional costs of debt for equity holders. Thus, debt financing has a negative effect on firm value because it strengthens the conflict between debt holders and equity holders.

Now consider both conflicts simultaneously. In this case, debt financing affects firm value both positively, by weakening the conflict between shareholders and managers, and negatively, by strengthening the conflict between debt holders and equity holders. The tradeoff between these costs and benefits implies that there is a level of debt that maximizes firm value. This value of debt determines an optimal capital structure in Jensen and Meckling (1976).

Other papers based on agency costs approach use at least one of the conflicts described by Jensen and Meckling as a starting point. We divide these papers into two subsections. In the first subsection we review papers that explore the conflicts between equity holders and managers; in the second subsection we investigate the asset substitution effect that arises from conflicts between debt holders and equity holders.

2.2.1 Equity holders vs Managers

Papers presented below investigate different costs and benefits of debt when there is a conflict between equity holders and managers.

In Stulz (1990) managers are empire-builders and always want to invest all the free cash flows they receive. They seek to make the firm as large as possible so long as the larger firm makes it easier to transfer the firm resources to manager's personal benefits. Consequently, when cash flow is high, they invest even in negative present value projects instead of paying out cash. In contrast, when the cash flow is low, managers do not have enough cash even for positive present value projects. They also cannot credibly convince shareholders to give them additional cash because it is always advantageous for them to pretend that some positive present value projects are unfinanced. By imposing debt on the firm, investors decrease the degree of over-investment in the states where investment would otherwise be too high - this is the benefit of debt. However, the use of debt can stop some of profitable investments in the other states of the world - this is the cost of debt. The tradeoff between the costs and the benefits of debt determines some optimal value of debt that maximizes the value of the firm in this model.

In Harris and Raviv (1990) managers always want to continue the firm's operations although sometimes liquidation is preferred by the investors. By imposing debt on the firm, investors create an opportunity to decide whether to liquidate the firm or continue the current operation. If the firm is not able to pay the debt, investors conduct a costly investigation to determine whether the firm should be liquidated.[†] This investigation not only decides the future of the firm, but also gathers useful information about the firm's ability to make future payments. Thus, the optimal

[†]With no debt investors can only liquidate randomly, while with positive debt they liquidate when firm defaults and information about the firm is bad.

amount of debt is determined by trading off the information and opportunity to liquidate, against the possibility to incur investigation costs.

In Jensen (1986) and Hart and Moore (1995), debt payments reduce free cash flows. While Jensen stresses the role of short-term debt in forcing managers to disgorge free cash flows[†], Hart and Moore examine the role of long-term debt in constraining self-interested managers from raising new capital.[†] The tradeoff in Hart and Moore (1995) is as follows. If the company has a small long-term debt, in some states managers will be able to finance some negative-net-present-value projects and there will be over-investment. On the other hand, if the company has a large long-term debt, in some other states managers will be unable to finance some positive-net-present-value projects and there will be under-investment. As usual, the optimal value of debt trades off the costs of debt against the benefits of debt.

To summarize, in this subsection we considered models that explore the conflict between equity holders and managers. This conflict arises because there is a separation between ownership and control. The following result describes benefits and disadvantages of issuing debt.

Result 2.1 *The benefits of issuing debt are*

- *increased managerial ownership (Jensen and Meckling 1976);*
- *negative present value projects are not financed (Stulz 1990);*
- *reduced free cash flow (Jensen 1986 and Hart and Moore 1995); and*
- *unprofitable firms are liquidated (Harris and Raviv 1990).*

The disadvantages of issuing debt are

- *asset substitution (Jensen and Meckling 1976);*

[†]Jensen (1986) does not specify a tradeoff, he emphasizes the benefits of debt and has little to say about the costs. There is no formal analysis in his paper.

[†]Once again, in Hart and Moore's model there are some projects that the managers seek to undertake even though they are not profitable. For such projects, the owners would prefer the cash to be returned to them, while the managers prefer to invest.

- *investigation costs in the states of default (Harris and Raviv 1990); and*
- *positive present value projects are unfinanced (Stulz 1990 and Hart and Moore 1995).*

2.2.2 Asset Substitution Effect

A paper presented below investigates different costs and benefits of debt when there is a conflict between equity holders and debt holders.

Diamond(1989) constructed a game-theoretical model that explored reputation considerations of firm owners and managers. In Diamond's model firm owners, who maximize combined projects' return on a some finite horizon, can choose in every period between investing in a risky negative present value and a safe positive present value project. Both projects must be financed by debt. Due to the asset substitution effect outlined above in this section, the risky project maximizes a one period return. Debt holders, who are aware of the firm's willingness to finance risky projects, also know the firm's history of repaying its debt, which constitutes the firm's reputation. The better is the firm's reputation the cheaper is the debt. The following equilibrium arises in this model. Initially, when the firms are young they prefer risky projects. Some of them survive, the others default. The surviving firms get older and prefer safe projects, because the opportunity costs of the risky projects are quite high: they might loose their reputation. A reasonable implication of this model is that younger firms have lower debt than older firms.

Result 2.2 *In Diamond(1989) the benefits of issuing debt arise from reputation considerations, while the costs are due to asset substitution.*

2.3 Asymmetric Information

This section explores models in which managers have some private information about their firms. There are three different streams of literature covered in this section. First, the pecking order literature started with Myers (1984) and Myers and Majluf (1984), that asserts firms always prefer debt to equity. The second area of literature suggests that a firm uses debt to signal its quality to outsiders. The final strand of literature uses the fact that because managers are risk averse, an entrepreneur's ownership share, signals the firm's quality to the outsiders.

2.3.1 The Pecking Order Theory

The notion of the Pecking Order Theory was introduced by Myers (1984). In short, it can be described as follows.

1. Firms always prefer internal finance.
2. If external finance is required, firms use the safest security first - they start with debt and only use equity as a last resort.

The theoretical foundation for the Pecking Order Theory was proposed by Myers and Majluf (1984). In their model there are two types of firms in the market. The type of every given firm is known by the insiders, while outsiders are only aware of the common probability distribution: with probability p the firm has value L, while with probability $1-p$ it has value $H > L$.† All the firms have access to a some profitable project with the investment costs of I and the net present value of R. They can pay the costs by issuing additional equity. Let us show that separating equilibrium can exist in which all L-type firms invest while all H-type firms do not invest. In this equilibrium everyone knows that if the firm invests it is of L type. Once the investment is made, old shareholders in the L-type firm give up a fraction $\alpha = I/(R+L+I)$ of their shares $(R+L+I)$ to new shareholders. Thus, the old shareholders of L type receive the payoff $(1-\alpha)(R+L+I) = R+L$ from investing and issuing equity, which makes them better off. If investment occurs in H-type firms, investors give up the same fraction $\alpha = I/(R+L+I)$ of their shares $(R+H+I)$. That is they receive the payoff $(R+L)(R+H+I)/(R+L+I)$. For some particular parameters, namely

†Both managers and current shareholders are insiders, while investors and potential shareholders are outsiders.

$$(H - L)\alpha > R, \tag{2.1}$$

the old shareholders of H-type firms are worse off from investing and issuing equity, that is the payoff $(R+L)(R+H+I)/(R+L+I)$ is less than H. Thus, the above mentioned equilibrium is possible with some particular parameter values. There is a number of important applications that follow from this simple model. First, it is usually better to issue a safe security rather than a risky one. In particular, external financing using safe debt is better than financing by risky equity. Second, when low-risk debt is unavailable some of positive NPV projects might not be financed. Third, the share prices may fall with announcement of new share issues. Specifically, before the announcement, the firm's market value is $pH+(1-p)(L+R)$. In contrast, if the firm issues shares its value becomes $L+R$. For parameter values satisfying inequality 2.1, $pH + (1-p)(L+R) > L+R$, which proves that after the announcement, the value of the firm decreases. On the other hand, if the firm issues safe debt to finance investment, stock price will not fall.

A number of researches have extended the basic Myers-Majluf idea. For example, Krasker (1986) generalized their model by making the issue size a continuous choice variable. (In Myers-Majluf model the firm raises either some amount of equity, known by all investors or none at all.) Krasker's main results are that: the stock price is decreasing with the issue size; and there is an upper bound for the amount of money that can be raised by issuing equity. This second effect is termed 'equity rationing'.

Narayanan (1988) finds another advantage of issuing debt. If there is asymmetry with respect to the value of the new project, some negative net present value projects will be financed. This happens because the only information about the new project available for the public is whether the project is taken. It makes the price of issued equity that finances the project to be the expected average price of all the projects that go forward. Even though firms finance negative net present value projects they can be more than compensated for these losses by issuing overpriced equity. Thus, all the projects with the net present value higher than some negative value are taken. Narayanan's model also strengthen some of Myers and Majluf results. It shows that even if debt is risky it is always better than equity.† This result arises because, risky debt is less overpriced than risky equity, and the cut-off level for the risky debt is higher.

To facilitate the overinvestment problem mentioned above Heinkel and Zechner (1990) use Myers's (1977) idea: initially outstanding debt makes investment less attractive (there is a punishment in the case of default) and increases the cut-off level. Thus, when firms can choose their capital structure prior to when the projects become available, some level of debt is voluntarily chosen. This restricts the firms from future overinvestment.

Brennan and Kraus (1987), Noe (1988), Constantinides and Grundy (1989) and Viswanath (1993) obtained results inconsistent with the pecking order theory.

Brennan and Kraus (1987) modify the example of Myers and Majluf (1984) in the following way. Two types of firms, L and H, have access to a some profitable project. Initially, both have some positive debt outstanding. A possible equilibrium proposed by the authors is that both types issue equity. Firms of H type finance the new project and pay its debt at face value, while firms of L type only finance the new project. Investors can distinguish between both types by observing their behavior. In this separating equilibrium there is no incentive for either of the types to imitate each other. The benefit of paying the debt is that issued shares are highly priced, while the cost is that the firms buy their debt at face value. The benefit is bigger (smaller) than the cost if the firm is of H-type (L-type). The difference arises because L types have a bigger risk incorporated in the face value of debt. These results differ from those of Myers and Majluf (1984). Firstly, there is no underinvestment in this story. Secondly, issuing equity by itself is not a bad signal. In fact, if it is followed by payment of the debt it can even be a good signal.

Constantinides and Grundy (1989) assumed that in Myers and Majluf's model the managers possess some shares of the firm. The true value of these shares is maximized with the management being able to issue any type of security and being

†Myers and Majluf (1984) consider only safe debt.

capable of repurchasing existing equity. In this set-up there is a fully separating equilibrium in which all the firms finance positive net present value projects by issuing securities that are not straight debt nor equity. This result contradicts the pecking order theory, because here firms do not finance the projects via internal funds or riskless debt even if they are able to.

Noe (1988) changed the example of Myers and Majluf (1984) so that there are three types of firms in the market S, L and H, where $S < L < H$. As before there are only two possible options for how they can finance the new project: either use debt or equity. In the equilibrium with two firms L and H,[†] firms of H type issue debt and firms of L type issue equity. Now if S type is added, it will be able to imitate both L and H, but imitating H is more profitable. Thus, in the new equilibrium types S and H issue debt, while type L issues equity. The type that issues equity does not have the lowest value. However, average quality of the firms that issue debt is higher than that of the firms that issue equity. Thus, the conclusion that the announcement of equity issue is a bad signal is consistent with Noe's model.

Viswanath (1993) consider the example of Myers and Majluf (1984) in a dynamic framework. The paper assumes a firm faces profitable projects not only now, but also in the future. If the firm finances the current project using internal funds then it will have to issue equity to finance the future projects. For some particular parameters it might be optimal to issue equity now and finance the future projects with internal funds. In contrast to the pecking order theory, in this model, equity issuing is not recognized by the market as a bad signal, contradicting.

Result 2.3 *The Pecking Order Theory, derived in Myers and Majluf (1984), states that debt is preferred to equity. However, this result can be invalidated in some cases if firms are allowed to have a wider range of financing choices. Thus, whether the Pecking Order Theory is relevant is an open question for empirical investigation.*

[†]See discussion of Myers and Majluf (1984).

2.3.2 Signaling with Proportion of Debt

To describe the research in this area let us outline the model in Ross (1977), who suggests that the manager of a firm, whose wages depend on the firm's current and future values, uses debt to signal the quality of the firm (known only to him) to the market. The dependance of the wage on the current value of the firm gives him the incentive to signal, while a penalty in the case of bankruptcy dissuades him from overstating its value. To see how this process works in somewhat more detail, consider the following model. Let the future return of a firm be distributed uniformly on $[0, t]$, where t is positive and unknown to the investors. The manager, who observes t, chooses value of debt d, to maximize his wage that depends on expected weighted average of value of the firm now and in the future. The value now is denoted by $V(d)$, while the value in the future is the expected return $t/2$ minus expected bankruptcy punishment $Lf(d,t)$, where L is the value of punishment and $f(d,t)$ is the probability of punishment as evaluated by the manager. Thus, the manager maximizes the following function

$$(1-\theta)V(d) + \theta(t/2 - Lf(d,t)), \tag{2.2}$$

where the parameter θ is a weighting of present versus future value. When $\theta = 0$ the manager maximizes the firm's present value, while when $\theta = 1$ he maximizes the firm's future value. In general, θ characterizes how much weight the manager puts on the firm's future value.

The model presumes that investors infer that $t = \alpha(d)$; this means that they discern the firm's type from the chosen debt level. The value of the firm is a half of t, $V(d) = \alpha(d)/2$. Substituting $V(d)$ in equation 2.2 and differentiating with respect to d gives

$$\alpha'(d) = \frac{2\theta}{1-\theta} Lf'_d(d,t). \tag{2.3}$$

If the investors are able to infer the firm's type correctly then $\alpha(d(t)) = t$, which being substituted in equation 2.3 produces the following differential equation

$$\frac{\alpha'(d)}{f_d'(d,\alpha(d))} = \frac{2\theta}{1-\theta}L. \tag{2.4}$$

Once we assumed that the future return of a firm is distributed uniformly on $[0,t]$, the bankruptcy probability $f(d,t) = d/t$ if $d < t$ and $f(d,t) = 1$ if $d \geq t$. When $d < t$ the solution to differential equation 2.4 is

$$d(t) = \frac{(1-\theta)t^2}{4\theta L} + c, \tag{2.5}$$

where c is a some constant that can be obtained from initial conditions.† As mentioned above, the parameters have to be such that $d(t) < t$. From equation 2.5 one can see that increases in the bankruptcy penalty decrease the debt level, other things being equal.

Now let us calculate values of debt and equity. The probability of default is

$$p = Prob\{x < d(t)|t\} = \frac{d(t)}{t}, \tag{2.6}$$

consequently levels of debt and equity are

$$D = (1-p)d(t) + pE\{x|x < d(t)\} = (1 - \frac{d(t)}{2t})d(t) \tag{2.7}$$

and

$$E = V - D = \frac{t}{2} - (1 - \frac{d(t)}{2t})d(t). \tag{2.8}$$

Differentiating D/E with respect to t verifies that the debt-equity ratio and firm value (or profitability) are positively related.

Poitevin (1989) also uses debt as a signal, but in a different set-up. He considers

†To understand why the manager chooses the debt value to satisfy equation 2.5 let us see the trade off he faces. His utility function is given by equation 2.2, which consists of the weighed sum of the firm's present value and its future value. With higher values of debt the first term is higher because investors discern the firm has a high value of t. On the other hand, with higher values of debt the second term is lower due to increased probability of default. The optimal value of debt, which is given by equation 2.5, solves this trade off.

two firms: one is an incumbent in some market and the other one is a potential entrant to this market. The marginal costs of the entrant is private information. Here, there is the following signalling equilibrium. Low cost entrants issue debt, while high cost entrants and the incumbent issue equity. The cost to an entrant of issuing debt is that the incumbent firm can start predation, which might result in bankruptcy of the entrant. The benefit of issuing debt is that market treats the debt issuer as a low cost firm and consequently assigns to this firm a higher value. Thus, the main result of this paper is consistent with the result of Ross (1977).

Result 2.4 *From signalling considerations the issuing of debt is good news to the financial market.*

2.3.3 Managerial Risk Aversion

The basic idea for papers in this subsection is that managers are risk-averse and can signal the quality of projects by firm's leverage. That is, they can signal how much risky equity they are ready to retain. In the model of Leland and Pyle (1977) an entrepreneur has access to a project that returns $\mu + \epsilon$, where μ is constant and known by the entrepreneur and $E[\epsilon] = 0$. The investors do not know the exact value of μ, rather, they only know its distribution.

The manager has to raise I of external funds from investors. There are two ways in which the entrepreneur raises the funds: via issuing debt D and via issuing equity. That is

$$I = (1 - \alpha)[V(\alpha) - D] + D, \tag{2.9}$$

where α is the fraction of the equity retained by the entrepreneur, and $V(\alpha)$ is the value of the firm as a function of α. The manager maximizes his expected utility of end-of-period wealth, where wealth is given by

$$W = \alpha(\mu + \epsilon - D) + (1 - \alpha)(V(\alpha) - D) + D. \tag{2.10}$$

The first term represents the entrepreneur's equity, while the rest is his cash. Here it is assumed that the funds raised externally are invested with riskless zero return. Thus, only equity is risky and by choosing α the entrepreneur adopts a suitable level of risk.

Differentiating the manager's utility with respect to α and substituting the equilibrium condition that $V(\alpha(\mu)) = \mu$, gives the first order conditions for α. Leland and Pyle show that the optimal level of entrepreneur's ownership share α is an increasing function of firm quality μ. Further, the authors prove that under some specific conditions the increase in α increases debt level D. Moreover, firms with larger debt also have a larger fraction of the equity owned by insiders and are of higher quality. The intuition for this result is as follows. Managers of higher quality firms can afford to choose a larger fraction of the equity, which gets less risky with firm's quality, and corresponds to larger debt values. This is the main result of their paper.

Result 2.5 *Firms with larger debt also have a larger fraction of the equity owned by insiders and are of higher quality.*

2.4 Industrial Organization Models

There are two different types of models in this section. First, the models that explore how debt can influences strategic interactions among competitors. Second, the models in which debt influences interaction with customers and suppliers.

2.4.1 Debt influences rivals

One of the initial papers in this area is Brander and Lewis (1986). The main mechanism of their model is based on the 'asset substitution effect', which was described in section 2.2. Consider a two period game, in which two firms firstly choose debt levels and then output levels. As usual, such a game can be solved by backward induction. For given levels of debt the firms compete in the standard Cournot quantity game in the output market. The authors show that with a higher level of debt the equilibrium output of the firm increases, while the equilibrium output of its rival decreases relative to the standard Cournot quantities.[†] The explanation of this effect might be as follows. Imagine you are playing a game of chess and you are several pieces down relative to your opponent. To survive (or even to win) you may have to pursue an aggressive strategy, which might even include sacrificing more of your pieces. Additional sacrificing will not deteriorate the result: the position is pretty much lost anyway. An aggressive strategy can make the outcome of the game more stochastic. Similar intuition applies to the effect of changes in output levels with the increase in debt level. In the bad states shareholders get nothing: they pay all the revenue to debt holders. In the good states they receive the difference between the revenue and the level of debt. Thus, when the level of debt is positive, shareholders care only about good states. If the level of debt increases the set of good states gets smaller: only in very good states can the firm pay something to shareholders. With higher levels of debt the average marginal profit of good states is higher (see the previous footnote). It means that the equilibrium output of a given firm is higher. Once output is higher, the equilibrium outcome of the rival is lower because the rival's reaction function does not change with the change in the debt level of the firm. Typical reaction functions are presented in Figure 2.1.

Now let us move to the first period in which firms choose levels of debt. Imagine one of the firms does not use debt. In this case the other firm has an incentive to use some positive level of debt, because it increases its output in the second period, and consequently enlarges its revenue. A drawback of this behavior is that the revenue of the first firm is decreased. Thus, when both firms have a choice of issuing debt they face a type of prisoners' dilemma. Joint profit is maximized if both do not use debt, while in the unique Nash Equilibrium both firms choose to have some positive debt.

[†]To obtain this result the authors assumed that the marginal profit is higher in good states than in bad states, that is the marginal profit is an increasing function with respect to some parameter that reflects the degree of the state of being 'good'. If this assumption is reversed then the result will be opposite.

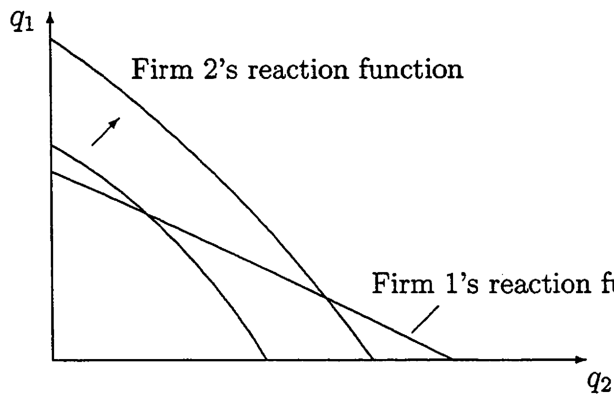

Figure 2.1: Typical reaction functions

From the finitely/infinitely repeated game literature, tacit collision over an extensive period is possible in oligopolistic industries. The monopoly level of profit can be achieved as a subgame perfect equilibrium for an infinitely repeated Cournot game if the following condition is satisfied, see for example Green and Porter (1984),

$$\pi_m \frac{1+r}{r} > \pi_d + \pi_c/r. \tag{2.11}$$

Here π_m is a half of the monopoly profit; π_d is the profit of a given firm if it deviates; π_c is a half of the Cournot equilibrium profit; and r is a discount factor. Thus, every firm has a strategy to play a monopoly until the rival deviates. After that the firm always plays an oligopoly. The condition states that the payoff of playing a monopoly for infinite period of time is higher than deviating now and playing oligopoly after that. In this story, shareholders maximize the value of the firm.

Maksimovic (1988) explores the case when shareholders maximize the value of equity, namely the value of the firm minus debt. Let the firm issue debt that promises to pay d every period. It is assumed that $d > \pi_c$, otherwise there is no effect from issuing debt, even in the case of Cournot equilibrium the firm is capable of paying the debt every period. If the firm always plays monopoly its payoff is $(\pi_m - d)(1+r)/r$. If it deviates the firm will get $\pi_d - d$ immediately, and nothing after that, as all the proceedings will go to debt holders. Thus, equation 2.11 is modified to

$$(\pi_m - d)\frac{1+r}{r} > \pi_d - d. \tag{2.12}$$

Maksimovic calls 'debt capacity' the maximum amount of debt that firms can handle without ruining the possibility of tacit collision. From the last equation the debt capacity is equal to

$$d < \pi_m - (\pi_d - \pi_m)r.$$

The author shows that the debt capacity decreases with the interest rate and increases with the elasticity of demand.

Result 2.6 *Consideration of strategic interactions among competitors results in a positive level of debt.*

2.4.2 Debt influences customers/suppliers

Titman (1984) argues that liquidation of a firm may impose some additional costs on its customers or suppliers. Specifically, if the firm produces durable goods, the maintenance costs of the goods will increase after the firm liquidates. The consumers anticipate the possible future increase in the maintenance costs and reduce their demand for the firm's products in the current period. In other words, these additional maintenance costs are indirectly imposed on the firm itself in the current period via reduction in the prices of the produced goods. Because of this, the liquidation policy that maximizes a firm's current value takes into account the costs, which it imposes on its customers in the future in the case of liquidation. However, this policy is not credible because in the future it is optimal for the firm to liquidate whenever its liquidation value exceeds its operating value. To restrict itself in the future, the firm voluntarily chooses some level of debt. If the level of debt is chosen appropriately, the firm is terminated when liquidation is consistent with the initial optimal policy.

A similar idea was used in Maksimovic and Titman (1991). Consider a firm that can produce either low or high quality goods (services).[†] The consumers are unable to recognize the quality of these goods until they consume the goods. If the firm has no financial problems, it supplies high quality goods to maintain a favorable reputation. In contrast, when it has financial problems, the firm supplies low quality goods. Even if we assume that the firm has no financial problems now, no-one knows what might happen to it in the future. As in Titman (1984), the value maximizing firm has time-inconsistent policies ex ante (when it has no financial problems) and ex post (when the firm gets some financial problems). Ex ante it is optimal to take into account possible future financial problems, while ex post it is optimal to supply low quality goods. To mitigate this problem, the firm restricts itself by issuing some level of debt ex ante. With a proper choice of debt, the firm's operations will be terminated whenever the firm will has financial problems.

Result 2.7 *Consideration of strategic interactions with customers and suppliers results in a positive level of debt.*

2.5 Corporate control considerations

Models using corporate control considerations are based on the fact that equity holders have voting rights while debt holders do not. This difference can have a crucial influence in the case of takeover attempts. To describe the research in this area we use the model of Stulz (1988).

In Stulz (1988) the optimal value of shares possessed by the management maximizes the expected gain to the investors from the takeover attempts. To outline his

[†]One possible example is the airline industry, where we observe such low quality airlines as Aeroflot and such high-quality airlines as Qantas.

model, let us consider the following simple set-up. There is a manager (who possesses fraction β of all shares), investors (who possess fraction $1-\beta$ of all shares), and a potential rival. Further, assume the manager never sells his shares, while the investors are distributed heterogeneously with respect to their reservation prices. The takeover benefit A to be acquired by the rival is unknown by the manager and investors, while its distribution function $F(A)$ is common knowledge. To acquire control, the rival has to buy at least 50% of all shares. It means that at least fraction

$$\alpha(P^*) = \frac{1}{2(1-\beta)} \tag{2.13}$$

has to be purchased by the rival from the investors, where P^* is the level of the premium paid to the investors.[†] Further, the probability of a takeover is just $1 - F(P^*)$, so the expected gain to the investors is

$$B = P^*(1 - F(P^*)). \tag{2.14}$$

The optimal value of β is chosen to maximize B where from equation 2.13

$$P^*(\beta) = \alpha^{-1}\left(\frac{1}{2(1-\beta)}\right). \tag{2.15}$$

To summarize, a larger β leads to a larger premium in the case of successful takeover, but a smaller probability of this takeover. As has been already mentioned in this chapter changes in leverage cause changes in the fraction of shares possessed by the manager. Thus, the main idea of Stulz (1988) is that the optimal debt/equity ratio maximizes the expected gain to the investors from possible takeovers.

Result 2.8 *The optimal capital structure maximizes the expected gain to the investors from possible takeovers.*

[†]The intuition for this equation is as follows. If α is an increasing function in P^* (When the premium is bigger, more investors are ready to sell their shares.) then P^* is an increasing function in β. It means that the bigger is the part of shares possessed by the manager, the harder it is to acquire 50% of all remaining shares.

2.6 Structure of debt contract

A recent contribution to corporate structure literature deals with incomplete contracts. Given its importance and size, this area of literature is a primary focus of this chapter.

Hart and Moore (1998) claim:

> ... economists do not yet have a fully satisfactory theory of debt finance (or the differences between debt and equity). One of the reasons for this is that debt is a security with several characteristics: a debtor typically promises creditor a noncontingent payment stream, provides the creditor with the right to foreclose on the debtor's assets in a default state, and gives the creditor priority in bankruptcy... (p. 1)

The second of these characteristics is explored in two papers: Hart and Moore (1994) and Hart and Moore (1998). Hart and Moore (1994) examine debt contracts when a debtor can repudiate the initial contract by withdrawing his human capital from the project. In their model the capital borrowed from the investor plus manager's wealth are invested by the manager at the beginning of the first period. This investment generates a stream of nonnegative returns $R(t)$, $0 \leq t \leq T$ and a stream of nonnegative liquidation values $L(t)$, $0 \leq t \leq T$. The timing of the model is presented in Figure 2.2. The returns offered to the investor may not be credible

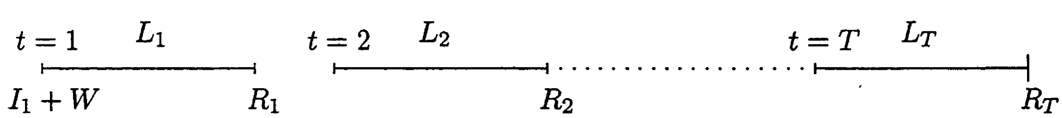

Figure 2.2: Time Line for the firm

as the entrepreneur cannot commit not to remove his valuable human capital from the project and renegotiate the investor's return.† If in some period the outcome of renegotiation leaves the investor with a combined return less than his investment minus returns he has already received, he would not be willing to finance the project ex ante. Thus, the main point of this paper is that only some investment projects that have large enough returns and liquidation values can be financed. The optimal debt contract structure here is designed to protect the investor from opportunistic behavior of the manager.

Hart and Moore (1998) consider a similar situation in which investment is provided by one party (an investor) and returns accrue to the other party (a manager). The manager can divert the project returns but cannot steal the assets used in the project. He promises to make a fixed stream of payments to the investor. As long as he makes these payments, he continues to run the project. If, however, the manager fails to pay, the investor recognizes that the manager diverts some project returns. At this stage the parties can renegotiate the initial contract. The following assumptions from Hart and Moore (1994) are used in this model. First, if the manager defaults, the investor is assumed to have an option to terminate the project. However, he uses this option only as a last resort. Second, the outcome of renegotiation for the investor is at least his return in the case of project's termination. Thus, if in some period the outcome of renegotiation is less than the capital he invested minus returns he received, the investor would not be willing to finance the project ex ante. Again, as in Hart and Moore (1994), the optimal debt contract in this model is designed to secure the investor from opportunistic behavior of the manager.

Another model that uses a very similar set-up is of Neher (1999). He explores the case of staged financing instead of up-front financing. In his model the capital is invested gradually and the returns are generated only when the project is completed. In all other respects, his model coincides with the model of Hart and Moore (1994).

†As the investor can terminate the project any period and receive that period's liquidation value, his minimum renegotiation return is the project's liquidation value. The exact value of the renegotiation return depends on the bargaining solution used. Nevertheless, the only that important here is that this return is increasing with the liquidation value.

The main result of Neher is that he derives the optimal staged investment path when the entrepreneur is unable to commit not to repudiate. Here, as in Hart and Moore (1994, 1998), the optimal debt contract is designed to secure the investor from the opportunistic behavior of the manager.

2.7 Conclusion

This chapter provided a selective review of corporate finance models and, in particular, their explanations of how firms choose their capital structure. Firstly, corporate structure is determined by agency costs. Secondly, presence of incomplete information influences the optimal debt level. Further, consideration of features of the theory of industrial organization also changes the debt/equity ratio. Finally, corporate control consideration is a determinant of corporate structure. As an extension to this literature we also considered a recent contribution to the capital structure theory. This literature provides insight into the optimal structure of debt using the incomplete contract theory approach.

CHAPTER 3

Staged Financing with a Variable Return

3.1 Introduction

Consider a cash constrained entrepreneur who has access to a profitable investment project with some rate of return s, that is constant and greater than one. If the entrepreneur had enough money he would buy physical assets worth K dollars and work with them for one period. At the end of this period he would receive a return $R = sK$ dollars that would completely cover the costs of his physical capital and labor investments. With no wealth, however, the entrepreneur has no choice but to find an outside investor to finance the project. The investor must be offered a large enough part of the return to at least completely cover his costs. Hart and Moore (1994) considered this basic set-up when the initial agreement between the entrepreneur and the investor is not enforceable. The entrepreneur may repudiate the initial contract by threatening to withdraw his human capital from the project. Anticipating such behavior from the entrepreneur, the investor will not finance some profitable projects.

To illustrate this point, let us consider the following example. The rate of return s is equal to 1.8 and total cost of capital investment K is equal to \$1. The entrepreneur borrows \$1 from the investor and promises to return \$1 to him in one period's time. After the investor lends the money the entrepreneur repudiates the initial agreement. We assume that both agents have equal bargaining powers. As the investment is sunk the investor will receive half of the final return, that is $sK/2 = \$0.9$, which is less than the total capital invested. Consequently, the investor will not finance such a project up-front.

To mitigate the problem outlined above, Neher (1999) assumed the entrepreneur stages the physical capital investment such that, for example, $K/2$ dollars are invested immediately, and the remaining $K/2$ dollars are invested in one period's time. Such a project returns $R = sK$ dollars in two periods. The outcome produced in the first period is used as collateral in the second period. This collateral reassures the investor that the entrepreneur will not repudiate the initial agreement in the second period. Also, first period repudiation will not occur because the entrepreneur's net return with repudiation is smaller than his net return specified in the initial contract. The investor may therefore be able to finance the project with the investment staged in two rounds, something he would not be willing to do if all the investment was provided up-front.

Let us go back to our example and show that it is possible to finance the project in 2 stages. Let the up-front investment be $I_1 = \$0.6$ and the investment in one period time be $I_2 = \$0.4$. The entrepreneur receives $R = \$1.8$ over the two periods and promises to pay back the investor \$1; the entrepreneur's net return is \$0.8. What if the entrepreneur repudiates the initial contract straight after I_1 has been invested? I_1 is sunk and the net return of the project is $R - I_2 = \$1.4$. As both parties have equal bargaining powers the entrepreneur receives $\frac{1}{2}(R - I_2) = \$0.7$ and the investor receives the remainder, that is $I_2 + \frac{1}{2}(R - I_2) = \$1.1$. The entrepreneur's return in the case of repudiation is smaller than his return specified in the initial contract. Therefore, he has no incentive to repudiate the initial contract in the first period. What if the entrepreneur repudiates the initial contract after I_2 has been invested? In this case the investor has an option to liquidate the project and receive the output from the first period. The return of the investor is $sI_1 = \$1.08$, which is greater than his costs. Again, the entrepreneur has no incentive to repudiate the initial contract. This shows that the investor can finance the project in two stages.

Unlike the example above, empirical and theoretical research asserts that the rate of return is not constant. In fact, returns may differ significantly over different stages of the project. For example, returns may decline due to the law of diminishing returns.[†] Non-constant returns also arise due to fixed costs.[†]

A non-constant rate of return - so that $s(\cdot)$ be a function of investment made up until that date - allows for an examination of a wider scope of projects including, for example, projects with fixed costs or increasing and decreasing rates of return. As shown below, relaxing the assumption of constant returns significantly alters the optimal investment path - this is the main result of this chapter. Although the algorithm of the optimal investment path derivation is quite similar to the algorithm presented in Neher(1999), some of the results presented here are considerably different. For instance, Neher (1999) predicts that positive wealth of the entrepreneur and a lack of discounting (very short time duration of one period) ensures that all profitable projects go forward. On the contrary, the model presented here shows that this is not the case. Unlike Neher, if the returns from the first periods do not produce large enough collateral for the future periods the project will not be financed, even if it is profitable.

Another difference arises with respect to the optimal investment path itself. Neher (1999) claims that starting from the second investment this path is always monotonically increasing. The result presented here is that the optimal investment path in general is not monotonical. In both models the values of investments are determined by collaterals, while collaterals are determined by rate of return functions. However, if the rate of return function is decreasing, the values of collateral for new investments may also be decreasing. In this case the sequence of investments will be decreasing as well.

This chapter finds an additional source of collateral for the investor. The outcome of repudiation in some periods might be termination of the investment project. If this threat is credible, the entrepreneur will never repudiate in these periods. We construct the optimal investment path with all available collaterals and prove that

[†]Madalla and Miller (1989) stated this law as: '...holding technology and the quantities of all other inputs constant, as equal increments of the variable input are employed, a point will eventually be reached where the increments to output begin to decline (p. 162).'

[†]Stiglitz (1993) describes fixed costs in the following way: 'First, there is the cost of the fixed input. To begin production, a firm may need space, machines, and a core of workers. The costs associated with fixed inputs are called fixed costs, or overhead costs (p. 316).'

the termination collateral is never used on the optimal investment path.

In this work we also explore the issue of repudiation in more detail. We construct a model in which both agents are able to repudiate. From this we find that allowing the investor to repudiate mitigates the commitment problem so much that all profitable two-period projects are financed.† There is still some inefficiency in this set-up with respect to financing in one period, but this inefficiency is much smaller than in the case when only the entrepreneur has the right to repudiate.

To address the issue of what the costs of staging are, we construct a model with costs of staging other than costs of delay. We assume, for example, that $K/2$ dollars are invested straight away and the remaining $K/2$ dollars are invested in a half period time. This project returns $R = sK$ dollars in one period, and the costs of staging are only due to chopping this period. Surprisingly, we find that the optimal investment path for this model exactly coincides with the optimal investment path for the main model with no discounting. We find that with respect to a model without any costs of staging, adding additional costs of 'chopping' the period only diminishes the likelihood that the project will be financed, but does not change the optimal investment path.

The model also generalizes several of Neher's results for projects with a variable rate of return. For example, an increase in the profitability of a project both increases the probability that the project will be financed, and weakly decreases the number of stages needed. We derive similar results with respect to a decrease in the discount rate, an increase in tangibility of the physical assets of the project, and an increase in growth of physical assets in outside value. Further, we relax the assumption that the entrepreneur has no wealth and find that it is optimal for him to invest all his wealth at the beginning of the project, before relying on outside financing in subsequent rounds.

The issue of commitment has been examined in a number of recent papers. Hart and Moore (1994) construct a model in which the capital borrowed from the

†There is a some condition on how the second period investment relates to the first period investment.

investor plus entrepreneur's wealth are invested by the entrepreneur at the beginning of the first period. This investment generates a stream of nonnegative returns $r(t),\ 0 \leq t \leq T$ and a stream of nonnegative liquidation values $l(t),\ 0 \leq t \leq T$. The returns offered to the investor may not be credible as the entrepreneur cannot commit not to remove his valuable human capital from the project and renegotiate the investor's return. If in some period the outcome of renegotiation leaves the investor with a combined return less than his investment minus returns he has already received, he would not be willing to finance the project ex ante. Solving for all possible repayment paths such that the entrepreneur has no incentive to repudiate, the authors show how repayment paths change with the maturity structure of the project return stream, and with durability and specificity of the project assets.

Neher (1999) used the same set-up to explore the case of staged financing instead of up-front financing. In his model the capital is invested gradually and the returns are generated only when the project is completed. He derived the optimal staged investment path when the entrepreneur is unable to commit not to renegotiate. He also explored how different parameters, such as the entrepreneur's wealth, project profitability and the tangibility of project's assets, affect the optimal investment path. He claims his predictions are found to be consistent with the recent empirical studies, see for example Gompers (1995). The crucial differences between Neher's and Hart and Moore's models are that in Neher (1999) the capital is invested in stages and the returns are generated only when the project is completed.

A paper very similar to the model presented here is Admati and Perry (1991), who explored the role of staged financing in overcoming a commitment problem. In their model, two players invest consecutively one after the other until the total investment exceeds some value, in which case both players receive some benefits. The crucial feature of their model is the presence of free-riding and costs of delay. The free-riding gives incentives for the players to invest less (in the hope that the other party will incur the cost instead), while the costs of delay push them to invest more (so as to receive the benefits of the project sooner than later). Although there is no collateral in their model, the results of their model concur with the basic intuition of our model: that is, some of the projects that cannot be financed up-front due to the commitment problem can be financed in stages.

Staged financing has been also explored in a number of papers. For example Gompers (1995), Sahlman (1988, 1990), Admati and Pfleiderer (1994), Bolton and Scharfstein (1990), and Roberts and Weitzman (1981). Staging there arises as a result of uncertainty or asymmetric information, rather than the commitment problem outlined above.

This chapter can be also considered as a contribution to the literature on the theory of capital structure. For excellent surveys of this literature, see Harris and Raviv (1991) and Hart (1995).

The rest of the chapter is organized as follows. The model is set out in section 3.2. Section 3.3 presents an algorithm for finding the optimal investment path. It also contains some basic extensions to the model and comparative statics. Section 3.4 extends the model to explore the situation when both agents are able to start renegotiation of the initial contract, while section 3.5 constructs a model with different costs of staged financing. Finally, section 3.6 concludes the chapter. Some of the proofs are contained in the Appendix.

3.2 The Model

We use the following model, first laid out by Neher(1999). Consider an entrepreneur, who needs to raise capital to finance a profitable project. The entrepreneur does not have any wealth and therefore must get an outside investor to finance the project. There is a competitive supply of deep-pocketed investors, who break even in equilibrium. These points are summarized in Assumptions (3.1) and (3.2).[†]

Assumption 3.1 *The entrepreneur has no wealth.*

[†]We relax Assumptions (3.1), (3.4), (3.5), (3.10) and (3.16) later in the chapter.

Assumption 3.2 *The supply of investors is competitive.*

Both physical assets and human capital are invested in the project. Physical capital is supplied solely from the investor in the form of an investment path $I_1, I_2, \ldots, I_T$, where the structure of the investment path - that is size of each investment I_t and the number of periods of investment T - is determined endogenously. This structure is chosen before the investment begins and cannot be changed afterwards. Further, the total amount of the physical capital investment is fixed and is equal to K, that is

$$\sum_{t=0}^{t=T} I_t = K. \tag{3.1}$$

Assumptions (3.3) and (3.4) outline these points.

Assumption 3.3 *The structure of the investment path is determined endogenously at the very beginning and cannot be changed afterwards.*

Assumption 3.4 *The total amount of the physical investment is fixed at K.*

Working with the physical assets, the entrepreneur invests human capital. There are no effort costs on the part of the entrepreneur. It is also assumed that he has no other income outside of the project. This is summarized in Assumptions (3.5) and (3.6).

Assumption 3.5 *The entrepreneur has no effort costs.*

Assumption 3.6 *The entrepreneur has no outside income.*

Each round of investment matches to a single period of the project and is structurally identical. The length of every period is fixed and is the same for all periods. The investment is made at the beginning of every period and is followed by possible renegotiation (the procedure of renegotiation is outlined below). At this stage the human capital is invested and finally output is produced. For details see Figure 3.1.

Both the entrepreneur and the investor discount the future with a some common per period discount factor $0 < \beta \leq 1$.

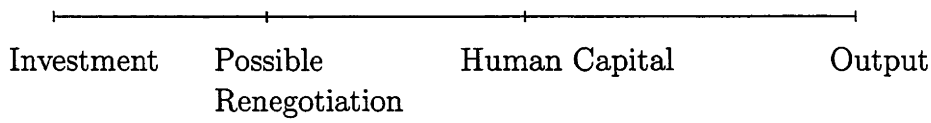

Figure 3.1: Sequence of events for every period

Assumption 3.7 *Discount factor β is the same for both agents.*

The next assumption is the main innovation of this chapter. We assume the rate of return s is some function of the whole investment made to date, that is $s : [0, K] \to \mathbf{R}$.[†]

Definition 3.1 $s(\cdot)$ *is a rate of return function of investment.*

The only requirements we impose on this function is that the integral $\int_0^K s(x)dx$ exists and is greater than K/β, ensuring profitability. This integral is the final return of the project, that is

$$R = \int_0^K s(x)dx. \tag{3.2}$$

Definition 3.2 R *is the final return of the project.*

No other returns can be generated in the project. The following assumption summarizes this discussion.

Assumption 3.8 *All projects are classified by rate of return functions $s(\cdot)$. In all other respects the projects are identical.*

In every period the project can be terminated during renegotiation, in which case the return of the project is the outside value of the assets in place at the time

[†]In Neher (1999) s is constant, therefore our assumption is more general.

of termination: this is equal to the value of the project without the human capital of the entrepreneur. This value is called the 'liquidation value' and is denoted by L_t, where $t = 1, 2, \dots, T$.

Definition 3.3 $L_t \; \forall \; t = 1, 2, \dots, T$ *is a liquidation value of the project in the case of its termination.*

We determine L_t in the same way as Neher(1999). Before the investment has had the whole period with the entrepreneur it has zero liquidation value, which means that the last piece of capital invested before termination is sunk.† All the previous investments have had at least one whole period with the entrepreneur, and they have the same value as at completion. Thus, in period t the liquidation value of the project is

$$L_t := \int_{x=0}^{\Lambda_{t-1}} s(x)dx, \tag{3.3}$$

where

$$\Lambda_t := \sum_{i=1}^{t} I_i. \tag{3.4}$$

In Figure 3.2 we draw rate of return $s(\cdot)$ as a function of the whole investment made to date. The liquidation value of the project is the area under this function.

The timing of the model is presented in Figure 3.3. At the beginning of every period t the entrepreneur receives a piece of investment I_t. During all this period the liquidation value is L_t. At the end of period T the final return R is realized.

In every period t after the investor puts I_t in the project, the entrepreneur might initiate renegotiation of the initial contract by refusing to work. His threat is to terminate the project.† This is a permanent end to the project - the project

†In the Appendix we examine an extension when only a part of the last investment is sunk. We also consider when the growth of the liquidation value comes not only via new investment but also changes exogenously.

†It is obvious that the entrepreneur never terminates the project. The outcome of the termination is that the investor gets the liquidation value and the entrepreneur gets nothing (for details see the discussion summarized by Assumption (3.12)). The entrepreneur only uses the termination threat as an incredible threat for the purpose of repudiation. All that important here, however, is that the mere threat by the entrepreneur triggers renegotiation. The investor, on the other hand,

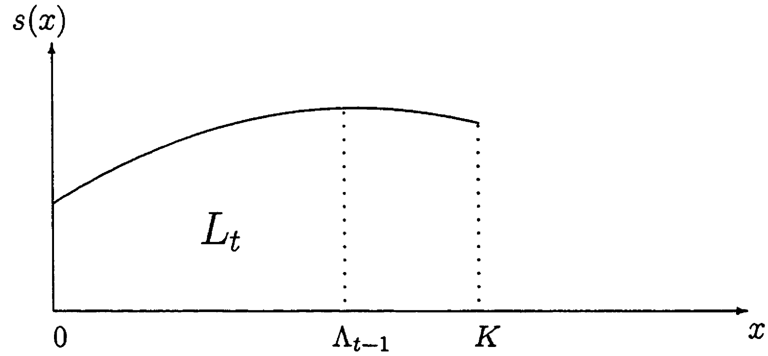

Figure 3.2: Rate of return function $s(\cdot)$ and liquidation value L_t

cannot be started again with some other entrepreneur. The following assumptions summarize this discussion.

Assumption 3.9 *The termination of the project leads to its permanent end.*

Assumption 3.10 *Only the entrepreneur can start the renegotiation process.*

If, on the other hand, the two sides find a way to resolve matters peacefully, the project will be continued. If the project is continued all the current physical assets remain in place, the investor stays with the project, and the next investments follow.

Assumption 3.11 *If, after renegotiation, the project is not terminated it starts again from the same place.*

We follow Hart and Moore (1994) and Neher (1999) by making the following assumption. As mentioned above in every period the entrepreneur has an option to

is assumed to have no option to start renegotiation of the contract. The situation when both the entrepreneur and the investor can start renegotiation will be considered in section 3.4.

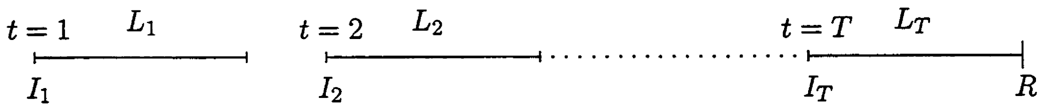

Figure 3.3: Time Line for the firm

repudiate the contract and make the conditions of the investor worse than in the initial contract. The worst penalty that the investor can impose on the entrepreneur is expropriating all available physical assets from the project. The investor cannot punish the entrepreneur by jailing him or by confiscating his personal belongings. Thus, the maximum the investor can receive during renegotiation from the entrepreneur is the liquidation value.

Assumption 3.12 *There is no outside project punishment for the entrepreneur.*

Here we have to make another very important assumption, that it is possible to verify who started the renegotiation process. If it were non-verifiable the investor could repudiate and claim that the process was initiated by the entrepreneur.[†]

Assumption 3.13 *Who initiates the renegotiation process is verifiable by a third party.*

Now, let us describe the renegotiation process itself. We denote by U_t, the present value to the investor of the outcome of renegotiation in period t, and by

$$S_t(T) := \beta^{T+1-t}R - \sum_{i=t+1}^{T} \beta^{i-t} I_i \tag{3.5}$$

[†]Neher(1999) also makes this assumption, although he does not make it explicit.

the surplus being bargained over in period t.†

Definition 3.4 $U_t \;\; \forall \;\; t = 1, 2, \ldots, T$ *is the present value to the investor of the outcome of renegotiation in period t.*

Definition 3.5 $S_t \;\; \forall \;\; t = 1, 2, \ldots, T$ *is the surplus being bargained over in period t.*

We assume that the entrepreneur and the investor have equal bargaining powers; that is, if they start bargaining over some surplus under equal conditions they will share the surplus equally.

Assumption 3.14 *Both agents have equal bargaining power.*

We also assume that an agent's outside option only affects the distribution of surplus, if it is optimal for the agent to adopt this option.

Assumption 3.15 *Outside options are unimportant unless they are binding.*

There is only one outside option for the investor. It is to liquidate the firm and receive the liquidation value. The entrepreneur has no outside options. Hence, in the case of renegotiation the investor chooses a half of the surplus if it is greater than the liquidation value. No termination occurs in this case because it worsens the payoffs for both players. (The outcomes in the case of termination in period t are none for the entrepreneur and L_t for the investor, while outcomes in the case of renegotiation are $S_t/2$ for both, where $S_t/2 > L_t$.)

If in some period t the liquidation value is greater than half of the surplus then there are two possible outcomes. If the liquidation value is less than the surplus itself, then by threatening to liquidate the firm, the investor increases his share up to the liquidation value. The termination does not occur, because with termination no-one is better. (The outcomes in the case of termination in period t are 0 for the entrepreneur and L_t for the investor, while outcomes in the case of renegotiation are $S_t - L_t$ for the entrepreneur and L_t for the investor, where $S_t > L_t$.)

† $\beta^{T+1-t}R$ is period's t value of the project's final return, while $\sum_{i=t+1}^{T} \beta^{i-t} I_i$ is period's t value of the investments to be made after period t.

If the liquidation value is greater than the surplus in some period t, the entrepreneur cannot entirely compensate the investor and persuade him not to liquidate the firm. It means that the outcome of repudiation in such a period t is always liquidation of the firm. The discussion above is summarized in the following result.

Result 3.1 *Termination is the outcome of repudiation in period t iff $S_t < L_t$ in that period.*

Proof. The proof follows from the discussion above and is omitted.[†] □

Definition 3.6 *If, for some investment path, termination is the outcome of repudiation in some period t, such a path is called an investment path with possible termination.*

Thus, for every $1 \leq t \leq T$ the present value to the investor of renegotiation in period t is equal to

$$U_t = max[L_t, S_t/2]. \tag{3.6}$$

Formula (3.6) is well known in the literature on non-cooperative bargaining with outside options, for example, see Section 3.12 of Osborne and Rubinstein (1990). This formula is similar to one used by Chiu(1998), Neher (1999), Muthoo (1999) and Hart and Moore (1994). It is based on an alternating-offers model with outside options of Shaked and Sutton (1984).

There is the following type of investment contract between the entrepreneur and the investor. This contract summarizes all the assumptions presented in this section.

Contract 3.1 *The investor provides all the rounds of investment in the initially specified investment path $I_1, I_2, \ldots, I_T$, and returns the future discounted value of this path at the end of period T. He does not get any revenue, he just breaks even.*

[†]See also discussion summarized by equation (3.9).

The entrepreneur works on the project for all T periods and receives the net return, that is the final return minus the payment to the investor. If the entrepreneur repudiates the contract in some period t, then the control over the physical assets is transferred to the investor. After that a new contract can be renegotiated or the investor liquidates the assets for L_t. If the new contract is specified, the project starts from the same place where it was before the entrepreneur repudiated. The investor provides the next rounds of investment and at the end of period T he receives some other value specified in the new contract.

An important aspect of this model is that there is an efficiency cost to financing the project via multiple rounds verses a single round. The final return R is the same regardless of the number of periods there is, but because of discounting there is a costly delay in realizing this return. On the other hand, the single round financing might not be feasible, because the entrepreneur might not be able to commit that he will not repudiate. As a result, the objective of the entrepreneur is to construct such an investment path that gives him the maximal net present value of the project. This path has to commit the investor at least to break even. In the next section we construct a procedure that finds the optimal investment path for the model outlined in this section.

3.3 Solving the model

Before proceeding, we outline the content of this section. Subsection 3.3.1 constructs a procedure that finds the investment path that is optimal among paths that cannot be possibly terminated. Subsection 3.3.2 considers paths with possible termination, and shows that the optimal path constructed in subsection 3.3.1 is optimal among all repudiation-proof paths, not just among paths without possible termination. In subsection 3.3.3 we present basic extensions and comparative statics.

3.3.1 Paths without termination

In this subsection we consider only investment paths without possible termination, that is paths with $S_t \geq L_t \quad \forall \quad t = 1, 2 \ldots, T$.

Assumption 3.16 $S_t \geq L_t \quad \forall \quad t = 1, 2, \ldots, T$.

The entrepreneur maximizes the following function with respect to the number of periods T and the investment path $\{I_t\}_{t=1}^{t=T}$

$$\beta^T R - \sum_{t=1}^{T} \beta^{t-1} I_t, \tag{3.7}$$

where $\beta^T R$ is the present value of the project return, and $\sum_{t=1}^{T} \beta^{t-1} I_t$ is the present value of the investment path.† Amongst the investment paths without possible termination, we restrict our attention only to repudiation-proof investment paths. On such paths, even if the entrepreneur initiates the renegotiation, he cannot make the investor worse off than in the case of the initial contract. Consequently, the entrepreneur does not have any incentive to trigger renegotiation along these paths.

Definition 3.7 *Repudiation-proof investment paths are such paths along which the entrepreneur will not trigger renegotiation.*

To solve for the repudiation-proof investment path we start from period T and move backwards period by period. The following repudiation-proof constraint is used in this model for period T

$$\sum_{i=1}^{T} \beta^{i-1} I_i \leq \beta^{T-1} U_T. \tag{3.8}$$

On the left-hand side we place the present value of the payment to the investor, as specified in the initial contract. On the right-hand side we display the present

†Note that for a given T, maximizing of the entrepreneur's utility function is equivalent to minimizing the present value of the investment path, that is the minimal feasible payment to the investor.

value of the payment to the investor in the case of repudiation in period T. The right-hand side consists of the present value to the investor of the outcome of the renegotiation in period T, plus the present value of the investment to be made by the investor in periods after T (for the last period the second term is equal to zero). The left-hand side must be not greater than the right-hand side for period T, otherwise the entrepreneur has an incentive to decrease the payment to the investor by repudiating.

Next, we move to period $T-1$ and construct a repudiation-proof constraint for that period. The only difference that arises between period $T-1$ and period T is that, in period T we do not need to be concerned with any future repudiation. In contrast in period $T-1$, the outcome of renegotiation might be affected by a possible future repudiation, namely in period T. If in period $T-1$ the investor anticipates that the entrepreneur repudiates in period T, and seriously bids down the investor's return, then in period $T-1$ the investor has a strong incentive to liquidate the firm. However, once the repudiation-proof constraint for period T is satisfied, the outcome in period $T-1$ is not affected by the outcome in period T, because the investor knows that the repudiation in period T will not occur. The repudiation-proof constraints for periods $t = 1, \ldots, T-2$ are constructed exactly in the same way as the repudiation-proof constraint for period $T-1$. All the repudiation-proof constraints are summarized by the following formula

$$\sum_{i=1}^{T} \beta^{i-1} I_i \leq \beta^{t-1} U_t + \sum_{i=t+1}^{T} \beta^{i-1} I_i, \quad 1 \leq t \leq T. \tag{3.9}$$

Again, the present value of the payment to the investor, specified in the initial contract, cannot be greater than the present value of the payment to the investor in the case of repudiation in period t, which consists of the present value to the investor of the outcome of renegotiation in period t, plus the present value of the investment to be made by the investor in periods after t.

After simplifying the inequality (3.9) we get

$$\sum_{i=1}^{t} \beta^{i-t} I_i \leq U_t, \quad 1 \leq t \leq T. \tag{3.10}$$

This inequality has exactly the same meaning as inequality (2) in Hart and Moore (1994). The pay-off to the investor in the case of repudiation must be at least as much as the expenses the investor has already incurred. In other words, investments should be completely covered by the liquidation value or by future return. If at some time t this condition does not hold, the entrepreneur has an incentive to repudiate the initial contract in that time. Because the final return for a given investment path is fixed, in the case of the repudiation the investor gets less, and hence the entrepreneur gets more than in the case of the initial contract. This observation gives some intuition why condition (3.10) has to be satisfied on repudiation-proof investment paths.

Now let us present the full maximization problem. For the given discount factor β, rate of return function $s(\cdot)$, and total amount of the physical capital investment K, the following utility function is maximized by the entrepreneur with respect to the number of periods T and investment path $\{I_t\}_{t=1}^{t=T}$

$$\beta^T R - \sum_{t=1}^{T} \beta^{t-1} I_t \longrightarrow \max_{T,\ \{I_t\}_{t=1}^{t=T}} \tag{3.11}$$

subject to (3.1), (3.2) and (3.10), namely†

$$\sum_{t=1}^{T} I_t = K, \quad R = \int_0^K s(x)dx \quad \text{and} \quad \sum_{i=1}^{t} \beta^{i-t} I_i \leq U_t, \quad 1 \leq t \leq T.$$

Before we present solution to this problem let us introduce a couple of definitions.

† As mentioned above in this subsection, we consider only investment paths that satisfy Assumption (3.16). We do not set such a requirement in problem (3.11) but rather derive a solution to this problem without Assumption (3.16) and show that the solution satisfies the assumption. Here we use the mathematical fact that a point which is optimal in some set will be optimal in any subset of this set that contains this point.

Definition 3.8 *Investment path $I_1, I_2 \ldots, I_T$ is feasible if and only if it satisfies conditions (3.1) and (3.10).*

Definition 3.9 *Investment path $I_1, I_2 \ldots, I_T$ is optimal if and only if it solves problem (3.11).*

The following proposition gives necessary and sufficient conditions for the optimal investment path.

Proposition 3.1 *A feasible investment path is optimal iff the following 4 conditions are satisfied*†

$$U_t = L_t, \ \ 2 \leq t \leq T, \tag{3.12}$$

$$\sum_{i=1}^{t} \beta^{i-t} I_i = U_t, \ \ 2 \leq t \leq T, \tag{3.13}$$

$$U_1(T) \geq I_1(T) \text{ and } U_1(T-1) \leq I_1(T-1), \tag{3.14}$$

and the Minimal Cost Condition MCC described below.

Minimal Cost Condition 1 *If there is only one investment path that satisfies conditions (3.12), (3.13) and (3.14), this path satisfies the MCC. If more than one investment path satisfies conditions (3.12), (3.13) and (3.14), there is a unique path that satisfies the MCC with the following pecking order property:*

- *of all the possibilities select the path with the lowest value of I_1;*
- *of all the possibilities that satisfy the condition specified in the previous item select the path with the lowest value of I_2;*
- *of all the possibilities that satisfy the conditions specified in the previous items select the path with the lowest value of I_3;*

and so on up until period T.

† *If $T = 1$ then condition $U_1(T-1) \leq I_1$ is unnecessary. Note also that from (3.3), (3.4) and (3.6) it follows that $U_1 = S_1/2$.*

The technical proof of proposition (3.1) can be found in Appendix. Here we present some intuition for conditions (3.12), (3.13), (3.14) and the *MCC*.

Condition (3.12) states that for periods $2 \leq t \leq T$, the present value to the investor of the outcome of renegotiation in period t is equal to the liquidation value in period t. It means that for the optimal investment path, the liquidation value (3.3) is equal or higher than a half of the surplus (3.5).† If in some period $t > 1$ the liquidation value (3.3) is less than a half of the surplus (3.5), then investment from periods $t = 1, 2, \ldots, t$ can be added together and invested in period 1. The constructed investment path will satisfy all the feasibility constraints (because the original investment path satisfies them) and will be shorter.

Condition (3.13) states that the feasibility constraints (3.10) for the optimal investment path in periods $2 \leq t \leq T$ are binding. If one of these conditions is not binding in period t, then investment from earlier periods (for example, from period $t-1$) can be moved to period t and this change will increase the entrepreneur's utility (same output, smaller costs), while all the repudiation-proof constraints stay satisfied (t constraint was non-binding and change is small enough to leave it non-binding, while all other constraints can only win from this change).

Condition (3.14) is constructed from the feasibility constraint (3.10) for $t = 1$. This condition states that the minimal T for which this constraint is satisfied is optimal. This is quite reasonable given the costs of delay.

The *MCC* chooses among the investment paths satisfying conditions (3.12), (3.13) and (3.14), a path that provides the smallest payment to the investor.† With the fixed outcome and fixed number of periods, it is optimal to wait and invest as late as possible. Thus, an analogue of the Pecking Order is generated by this condition.

Now let us construct an algorithm for finding the optimal investment path: this investment path solves problem (3.11). For a moment assume that T is given. From

†See equation (3.6).

†As was noted earlier, when T is optimal the problem of maximizing the entrepreneur's utility (3.11) is equivalent to minimizing the payment to the investor.

conditions (3.12) and (3.13) we derive

$$I_t = L_t - \sum_{i=1}^{t-1} \beta^{i-t} I_i, \quad 2 \le t \le T \tag{3.15}$$

that depends only on a choice of I_1.† Next, we use equation (3.1) and find the value of I_1. The value of I_1 solves

$$I_1 + I_2(I_1) + \ldots + I_T(I_1) = K. \tag{3.16}$$

If there is a non-uniqueness with respect to I_1 then the *MCC* is applied. Further, we use equation (3.15) and the *MCC* to derive the unique values of investments for $2 \le t \le T$. Thus, for any given T we can construct an investment path that is optimal when the number of periods is T. The next task is to find the optimal value of T. Condition (3.14) solves this task. It investigates whether the given T is optimal. The following algorithm summarizes this discussion.

Algorithm 3.1 *To derive the optimal investment path assume first that* $T = 1$. *Check condition (3.14). If this condition holds then the project is financed in one period with* $I_1 = K$. *If this condition does not hold then assume* $T = 2$. *Using equations (3.15), (3.16) and the MCC, derive the two-period investment path. Check condition (3.14). If this condition holds then the project is financed in two period. If not, then assume* $T = 3$, *etc.*

Let us illustrate Algorithm 3.1 in the following example.

Example 3.1 *Let* $\beta = 1$, $K = 5$ *and* $s(x) = 0$ *for* $x \le c$, $s(x) = 2$ *for* $c \le x \le K$, *where* c *is a some constant from* $[0, K]$, *see Figure 3.4. What is the maximal value of* c *for which the project will be financed?*

First, note that the project will not be financed in one period for any positive c. *Condition (3.14) is not satisfied because* $U_1 = S_1(1)/2 = \int_0^K s(x)dx/2 = (K - c)$ *is always less than* $I_1 = K$.

†The value of every next investment depends only on values of previous investments. For a given value of I_1 we can find I_2, then I_3 and so on.

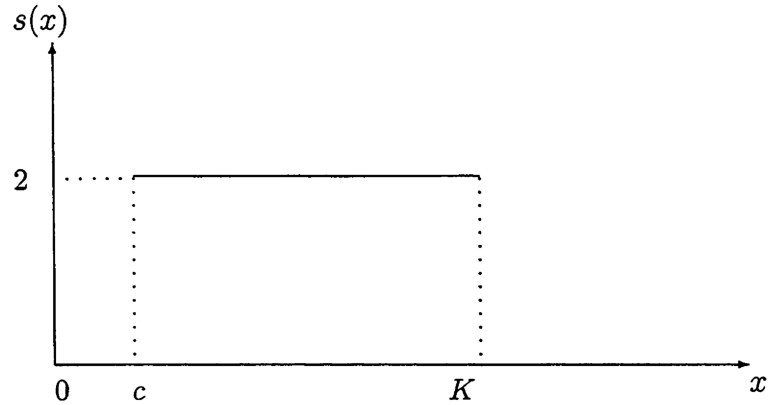

Figure 3.4: Rate of return $s(x)$

Second, assume that $T = 2$. Using equations (3.15), (3.16) and the MCC construct an optimal investment path. Equation (3.15) is $I_1 = L_2 - I_2$, while equation (3.16) is $I_1 + I_2 = K$. Combine them and derive $K = L_2$, where by definition $L_2 = \int_0^{I_1} s(x)dx = 2(I_1 - c)$. Thus, $I_1 = K/2 + c$ and $I_2 = K/2 - c$. Now we apply condition (3.14), that is $2U_1(2) = S_1(2) = \int_0^K s(x)dx - I_2 = 2K - 2c - I_2 \geq 2I_1$, which means that $c \leq K/6$. Thus, $c \leq \frac{5}{6}$ is the condition for the project to be financed in two periods.

Next, let T be a some large number. Using the same technique[†] *one can get that at the optimum $I_1 = K - 2c$, $I_2 = K - 4c$, $I_3 = 2I_2$, $\dots$ $I_t = 2I_{t-1}$ $\forall$ $t \in [3, T]$. It means that if $I_2 > 0$ then $\sum_{t=1}^{\infty} I_t = \infty$, and there exists some finite T that $\sum_{t=1}^{T} I_t \geq K$, that is the project is financeable.*

Thus, $c < \frac{K}{4} = \frac{5}{4}$ is the condition for a project to go forward. □

If the project is financeable the algorithm will find the solution. However, it is still unclear whether this solution fits the entrepreneur. The following result states that the entrepreneur always gets a positive final return on any optimal investment

[†]*See Appendix for details.*

path.

Result 3.2 *The optimal investment path constructed by Algorithm 3.1 delivers a positive final return to the entrepreneur.*

Proof. The proof is presented in the Appendix. □

Thus, the entrepreneur will always implement the outcome of Algorithm 3.1, because this outcome delivers a positive final return to him. As we mentioned in section 3.2 there are no outside options for the entrepreneur.

Addressing our discussion at the beginning of this section, the next result states that Assumption (3.16) always holds on the optimal investment path.

Result 3.3 *Assumption (3.16) always holds on the optimal investment path.*

Proof. The proof is presented in the Appendix. □

Thus, in this subsection we have constructed an algorithm that finds the investment path that is optimal among all feasible investment paths for which Assumption (3.16) is satisfied. We also prove that the entrepreneur is better off with this investment path. The objective of the next subsection is to explore paths for which Assumption (3.16) is not satisfied.

3.3.2 Paths with possible termination

To illustrate that paths with possible termination exist, let us consider the following example. Imagine that the first period investment $I_1 = \$1$ gives the return $R_1 = 10I_1 = \$10$, the second period investment $I_2 = \$2$ returns $R_2 = I_2 = \$2$ and the third period investment $I_3 = \$6$ returns $R_3 = 0.5I_3 = \$3$. The total capital investment is $K = \$9$, the total surplus $R = R_1 + R_2 + R_3 = \$15$, and the discount factor $\beta = 0.9$. Let us show that such a path is repudiation-proof and that $S_2 < L_2$. That is, repudiation in the second period does not occur, because its outcome is termination of the project, and that outcome does not suit the entrepreneur. First, the surplus bargained in period 1 is $S_1 = \beta^3 R - \beta I_2 - \beta^2 I_3 = \$4.275$. Half of this surplus is enough to cover the first period investment $I_1 = \$1$. Second, the surplus bargained in the second period is $S_2 = \beta^2 R - \beta I_3 = \$6.75$, which is less than the second period liquidation value $L_2 = R_1 = \$10$. Third, the liquidation value in the third period is $L_3 = R_1 + R_2 = \$12$, and it is greater than $I_1/\beta^2 + I_2/\beta + I_3 \approx \$9.46$. The constructed investment path is repudiation-proof.

Thus, there is an issue whether it is possible to improve the optimal investment path constructed in the previous subsection by taking into consideration investment paths with possible termination. The following result clarifies this issue.

Result 3.4 *The solution derived with the help of Algorithm 3.1 is optimal among all repudiation-proof paths independent on whether Assumption (3.16) holds along them or not.*

Proof. The proof is presented in Appendix. □

Result (3.4) allows us to derive the solution to the model outlined in the previous section for a general case (no other assumptions are required). For a given discount factor β, rate of return function $s(\cdot)$ and total amount of the physical capital investment K, Algorithm 3.1 constructs the optimal investment path, if such a path exists.

3.3.3 Basic results, extensions and comparisons

First, let us analyze the optimal investment path. Using equations (3.5) and (3.15) it is straightforward to derive that the value of entrepreneur's utility function (3.7) is equal to

$$S_1 - I_1 = \beta(S_2 - L_2) = \ldots = \beta^{T-1}(S_T - L_T). \tag{3.17}$$

The net present value of the surplus from continuing the project in every period t is equal to the entrepreneur's return. If the entrepreneur's return is lower in some period t, then the investor has an incentive to repudiate in that period and increase his return. On the other hand, if it is higher, then the project is not optimal, because it could be financed more quickly with smaller costs of delay. Thus, on the optimal investment path in every period $t > 1$, the net present value of the difference between the future surplus and collateral is the same, and equal to the final return of the entrepreneur.

The optimal investment path is always efficient after the agents sign the contract, because after the contract is signed both agents behave so that they maximize the total surplus of the project: the entrepreneur never repudiates; even if he repudiates the investor never liquidates the firm. Both repudiation and liquidation are inefficient, because $L_t < S_t \;\; \forall \;\; t = 1, 2 \ldots, T$.[†]

Second, consider some basic extensions. Let us allow the entrepreneur to have wealth greater than zero. This change produces two results.

Result 3.5 *It is optimal for the entrepreneur to invest all his wealth at the very beginning.*

Proof. The proof is presented in Appendix. □

Result 3.6 *Some of the profitable projects will not go forward even if an entrepreneur's wealth is greater than zero.*

Result 3.6 is different from the analogous result of Neher (1999), who found in his less general model that all the profitable projects go forward if the entrepreneur's wealth is greater than zero. To illustrate the last result let us consider example 3.1 from section 3.3. Let c in that example be equal to 2 and let the wealth of the entrepreneur ω be equal to 0.5. The project is profitable because its return is $R = 2(K - c) = \$6$, while its costs are $K = \$5$. Let us show that the project does not go ahead. From result 3.5 we know that the optimal strategy of the entrepreneur is to invest all his wealth at the very beginning. It means that the investor needs to finance only $K' = \$4.5$, and only $c' = \$1.5$ of these costs are fixed. From example 3.1

[†]Note, however, that before the contract has been signed there is some inefficiency due to costs of delay. We mentioned this inefficiency in the end of the previous section.

we know that the projects are financed if and only if $c' < \frac{K'}{4}$. This condition does not hold here, which means that the project does not proceed.

Now let us relax Assumption (3.4), namely the total amount of the physical investment is fixed at K. In general, we can assume that the total amount of the physical capital investment is only bounded from above as in Neher (1999)

$$\sum_{t=1}^{t=T} I_t \leq K \tag{3.18}$$

or not bounded at all

$$\sum_{t=1}^{t=T} I_t \in (0, \infty). \tag{3.19}$$

To find the optimal value of the physical capital investment we solve another optimization problem. Namely, for all feasible values of the physical capital investment we construct optimal investment paths, and find among them such a path that maximizes the entrepreneur's utility. In other words, we optimize optimal investment paths with respect to the physical capital investment. Thus, this set-up allows us to find an endogenous value of the physical capital investment.†

Let us illustrate this extension in the following example. Imagine that the investment project is construction of a building and the last piece of investment is painting this building red. Potential buyers of this building do not like red and will repaint the building yellow. It means that painting itself has negative productivity. The activity provides no returns, but has some positive costs. The optimal strategy for the entrepreneur is to sell the building unpainted, which can be regarded as maximization with respect to the the physical capital investment.

Next, let us focus solely on the optimal investment path. Is it always monotonically increasing? The answer is 'no'. In the Appendix we give an example of an optimal investment path with decreasing values of investments. Again, this result

†Note, that Neher (1999) always gets either the whole K is invested, either the project is not financed. The reason why the solution for our model is more complicated is because the increase in K does not always mean the increase in the entrepreneur's utility. For example when $s(\cdot)$ is some continuous decreasing function from some positive value to zero it is not optimal to invest the whole K. The segment where $s < 1/\beta$ by itself has a negative productivity.

is different from the finding of Neher (1999), derived for the constant rate of return case.† The explanation for the difference is as follows.

In the general case the values of investments are determined by collaterals, while collaterals are determined by rate of return functions. As a rate of return function can be such that the values of collaterals for new investments are decreasing, the sequence of investments for such functions will be decreasing as well.† For an economic reasoning, we can think about some seasonality. In Summer there is positive productivity, while in Winter the productivity is negative. It means that after Winter the value of collateral for new investments decreases, and the value of the next investment after Winter will be smaller than the investment in Winter. Another example of this is the case of Federal Express, discussed in Gompers (1995): the first investment in Federal Express was £12.25 million, the second was £6.4 million and the third was £3.88 million. Clearly, the sequence of investments is decreasing.

Further, another result, which is different to an analogous result of Neher (1999), is described as follows. Neher claims that with $\beta \to 1$ all profitable ventures become financeable. With more general return functions, as presented here, this is no longer true. For instance, in example 3.1, if $c = 2$ the project will not be financed for any T and for any β. However this project is still profitable, because $2(K - c) > K$. The reason why the project is not financed is because, the returns are very bad at the beginning of the project, and such bad returns do not produce large enough collaterals to secure next period's investments.

Finally, in the Appendix we present conjectures to the extensions of Neher (1999). First, we show that an increase in both the discount factor and rate of return function improves the optimal investment path. Second, we find that an increase in tangibility of the physical assets improves the optimal investment path. Third, we prove that an increase in growth of physical assets in outside value also improves the optimal investment path.

†Neher (1999) has a monotonically increasing sequence of investments starting from the second investment; that is, every next investment after the second one is bigger than the previous investment.

†In the constant rate of return case these values are always increasing, otherwise the project is unprofitable.

3.4 Repudiation of both agents

In this section we consider the situation when both agents are able to start renegotiation of the initial contract.† The investor can initiate renegotiation by refusing to contribute the previously agreed round of investment, while the entrepreneur as usual can trigger the renegotiation process by refusing to work.† Thus, only for purposes of this section we substitute Assumption (3.10) by the following assumption.

Assumption 3.17 *Both the entrepreneur and the investor can start the renegotiation process.*

Before we proceed we would like to return to our original example and show some reasons why we have to put an additional assumption in the case of this extension. Let rate of return s be constant and equal to 1.8, total cost of capital investment K be equal to \$1, and discount factor β be equal to 1. In the introduction we showed that when only the entrepreneur can repudiate, the investment path $I_1 = \$0.6$ and $I_2 = \$0.4$ is feasible. Let us show that this path is not feasible when both agents have the right to repudiate and all the other assumptions of the main model are the same. Imagine that after the output of the first period is released, but before the second period investment is made, the investor initiates repudiation and triggers liquidation. In this case his cost is just $I_1 = \$0.6$, while his return is $s * I_1 = \$1.08$. The net return for him is \$0.48, which is higher than zero return specified in the initial contract. Thus, now we are concerned not only about opportunistic behavior of the entrepreneur, but also about opportunistic behavior of the investor.†

Although the opportunistic behavior of the entrepreneur is very complicated†, the opportunistic behavior of the investor is easy to work out with the help of the

†The third and the last situation when only the investor is able to initiate renegotiation process is trivial. In this case there is no threat for the investor and all the profitable projects are financed in one period.

†Neher (1999) describes this extension in the introduction and claims that the results for it are exactly the same as in his main model. However, here we show that the results are qualitatively different.

†Both types of opportunistic behaviors are explained in details in Chapter 1 of Hart (1995).

†A lot of research has been devoted to this topic. See, for example, Hart and Moore (1994) and Neher (1999).

following assumption. Let us assume that during the time after the new output is released, and before the next investment is made, the victim of repudiation has all the property rights with respect to the new output.[†] The investor, if he is not the victim, gets these property rights only after he invests the next piece. Thus, the liquidation value for the time after the new output is released, and before the next investment is made is as follows

$$L'_t := \begin{cases} L_{t-1} & \text{when the investor repudiates,} \\ L_t & \text{when the entrepreneur repudiates.} \end{cases} \tag{3.20}$$

In Figure 3.5 we present the sequence of events and liquidation values for period t.

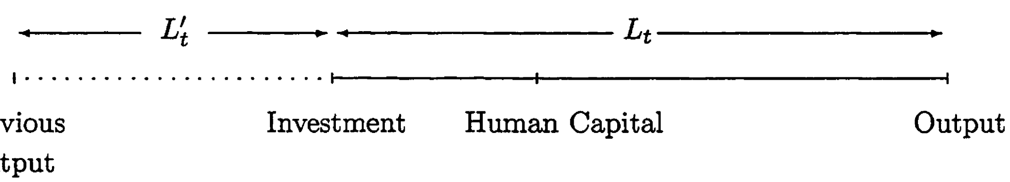

Figure 3.5: Sequence of events and liquidation values for period t

The following result with respect to this set-up holds.

Result 3.7 *Both agents might repudiate only straight after the next piece of investment is supplied.*

Proof. The proof is presented in Appendix. □

The utility function of the entrepreneur in the case when both agents can repudiate is the same as in the main model, specifically

$$\beta^T R - \sum_{t=1}^{T} \beta^{t-1} I_t. \tag{3.21}$$

[†]The victim of repudiation is an agent who does not initiate the repudiation process.

The only difference arises with respect to the constraints. Because both the entrepreneur and the investor can initiate the renegotiation process in the last period, the constraint with respect to the last period has to be binding. That is

$$\sum_{t=1}^{T} \beta^{t-T} I_t = U_t. \tag{3.22}$$

All the other constraints (3.10) do not appear in this extension, because even if one of the agents starts renegotiation in periods $t = 1, 2, \ldots, T-1$, the other one will have a chance to renegotiate the contract again in the last period. The following proposition, proved in the Appendix, determines the optimal investment path in the case of this extension.

Proposition 3.2 *When both agents can repudiate, the optimal investment path always has two periods. This path is a solution of (3.21) under (3.22).*

The collateral of the investor in the first period comes about due to his possibility to renegotiate the result of the first-period repudiation later (in the second period). The collateral in the second period is the same as in the main model, that is the output produced in the first period. Surprisingly, Proposition 3.2 shows that all profitable two-period investment paths that solve (3.21) under (3.22), can be financed.

This extension to the model allows the investor to counteract the entrepreneur's right to repudiate. Hence it helps to get the project financed more efficiently. In Figure 3.6 we present a graphical illustration of the efficiency improvement in the constant rate of return case. The horizontal axis depicts projects with different rates of return s, while the vertical axis depicts different discount factors β.

Area A includes all the projects that are financed in one period, while area B includes only the projects that are financed in two periods, and area C includes only the projects that are financed in three periods. We do not show the projects that are financed in more than three periods, because the figure becomes too difficult to illustrate. However, it can be seen from Figure 3.6 that area B is smaller than area A, area C is smaller than area B. That is, every next corresponding to more periods

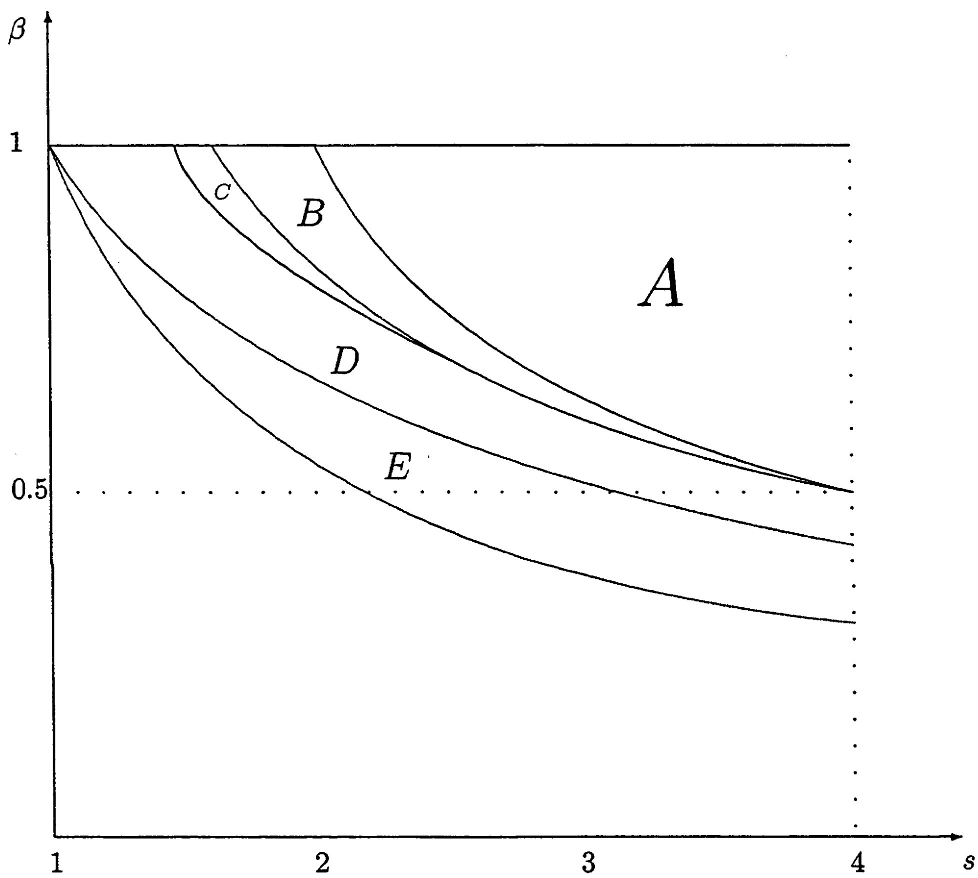

Figure 3.6: Illustration to the extension when both agents can repudiate

area is less than the previous one. Area $A + B + C + D$ represents all the projects financed when both agents can repudiate, while $A+B+C+D+E$ represents all the profitable projects. Thus, even in the case when both agents are able to repudiate there are still some efficiency losses, shown in Figure 3.6 as area E. The efficiency improvement is area D, minus the projects that are financed in more than three periods.

In this section we considered the extension to the main model when both agents are able to start renegotiation of the initial contract. We found that there is some efficiency improvement in comparison with the main model in this case. In the next section we examine an extension when the costs of staged financing are different from the costs described in the main model.

3.5 Different costs of staged financing

In this section we consider another extension, which explores a model with different costs of staged financing. We assume that the entrepreneur instead of financing in many periods chops the initial period into a number of small periods. On the one hand, there is no costs of delay from this extension. Up-front financing and staged financing both finish in one period time. On the other hand, there are some costs associated with this chopping procedure. These costs are gross costs and they also incorporate differences in discounting within a period. We call these costs $c(T)$, where $c'(T) \geq 0$ and $c''(T) \leq 0 \quad \forall \quad T \geq 1$.

The following example illustrates how such an outcome may occur in practise. Imagine that in the first Best case a project can be financed in 4 years, while in the Second Best case it can be financed in 16 years.[†] The decision to meet once a year will incur additional costs, but will probably be better than the decision to extend the production process across the 16 years.[†]

[†]We use the term First Best to refer to the solution without the hold-up problem, and the term Second Best to refer to the solution given in section 3.2.

[†]However, this decision is possible only when the entrepreneur can finish the first part in the first year, the second part in the second year, etc and this is different from what is assumed in the main model. This extension is 'another corner' and the real situation is somewhere in between.

The utility function of the entrepreneur in the case of this extension is as follows:

$$R - \sum_{t=1}^{T} I_t - c(T) = R - K - c(T). \tag{3.23}$$

It means that maximizing the entrepreneur's utility function is equivalent to minimizing the number of periods T. This result is also derived in the main model, but here it is much more straightforward.

The repudiation-proof constraints in this extension are as follows:

$$\sum_{i=1}^{t} I_i \leq U_t, \quad 1 \leq t \leq T. \tag{3.24}$$

Next step is to realize that we are in the conditions of the main model with $\beta = 1$.[†] We construct a graphical interpretation of Algorithm 3.1 in this case.

From the equations 3.1, 3.3 and 3.15 it follows that

$$K = \sum_{t=1}^{t=T} I_t = L_T = \int_{x=0}^{\Lambda_{T-1}} s(x)dx. \tag{3.25}$$

Graphically it means that I_T is determined in such a way that the integral of $s(x)$ from 0 to $\Lambda_{T-1} = K - I_T$ is equal to K, see Figure 3.7.

Knowing the value of I_T, we find the value of I_{T-1} in the following way. This value has to be such that the integral of $s(x)$ from $\Lambda_{T-2} := K - I_T - I_{T-1}$ to $\Lambda_{T-1} = K - I_T$ is equal to I_T, see Figure 3.8. Moreover, it follows that $A + B = K$ and $A = \Lambda_{T-1}$, which means that $B = I_T$. Thus, the procedure of finding I_{T-1} is the same as the procedure of finding I_T with the the total amount of the physical capital investment being $K - I_T$.

This algorithm can be continued. In the same way as we have constructed I_{T-2}, we can construct I_{T-3}, then I_{T-4} and so on. The criterion for finishing this

For example, it might be the case that every part by itself can be finished not quicker than in 2 years, which means that the project can be finished not quicker than in 8 years.

[†]The only difference is that some projects that are financeable in the main model will not be financed here, because here there are the additional costs $c(T)$.

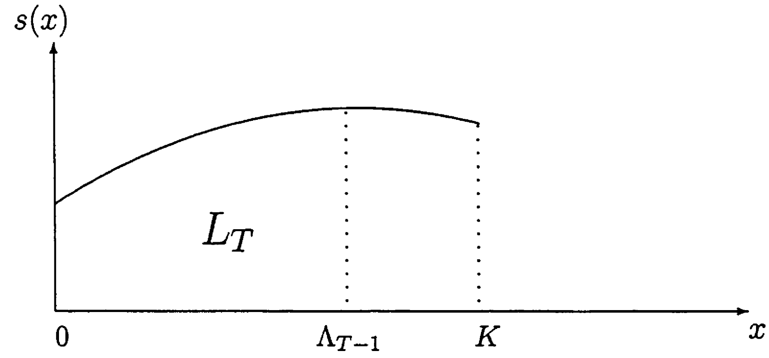

Figure 3.7: Illustration to graphical interpretation: construction of I_T

algorithm is as follows. Since the minimal feasible T is optimal, for every T we check whether $S_1(T)/2 \geq I_1$. Thus, before we construct I_T, we check whether the project is financed in one period. After we construct I_T, we check whether the project is financed in two periods, or Λ_{T-1} is financed in one period, that is $S_1(2)/2 \geq I_1 = \Lambda_1$. After we construct I_{T-i}, we check whether the project is financed in $i+2$ periods, or in other words $\Lambda_{T-i-1} := K - I_T - \ldots - I_{T-i-1}$ is financed in one period. That is $S_1(T)/2 \geq \Lambda_{T-i-1}$, where $T = i+2$.

There are two possible outcomes of this algorithm:

1) At some stage we finish the procedure, which means that we have found a candidate for the optimal investment path. If this candidate gives a higher outcome than $c(T)$ then the project can be financed.

2) We cannot finish the procedure, which means that this project cannot be financed.

This observation concludes the graphical interpretation.

One very strong conclusion follows form this graphical interpretation. The optimal investment path does not depend on the form of the cost function $c(T)$. The higher cost function can only make projects non-financed. Such a strong result is due to the assumption that $1/T$ of the period is enough to implement one round of

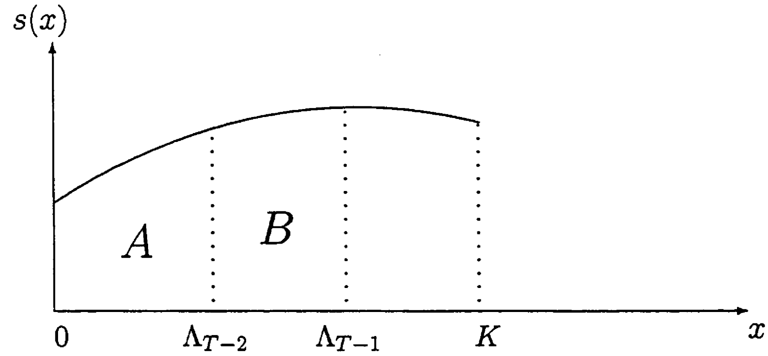

Figure 3.8: Illustration to graphical interpretation: construction of I_{T-1}

investment.

In this section we considered tan extension to the main model, where there are costs of staged financing due to the chopping of investment period. We found that this extension is equivalent to the main model when there is high enough outcome and no discounting. We used a graphical interpretation to illustrate the solution to this extension.

3.6 Conclusion

The main results of this chapter are as follows. First, we construct a solution to a generalized model of Neher (1999). Instead of a constant rate of return function we consider a variable one. Algorithm 3.1 describes the solution.

Second, we find that there exist repudiation-proof investment paths on which the outcome of repudiation in some periods is termination of the investment project. Such an outcome is not suitable for the entrepreneur and because of that he never repudiates in these periods. This can be considered as another source of collateral for the investor. This type of collateral has not been mentioned previously.

Third, we find the optimal investment path for a general problem with all types of collateral available and show that this path does not use termination as a collateral.

Fourth, we get the following results that are different from the results of Neher (1999). Neher (1999) found that all profitable projects become financed either without discounting or with positive wealth of the entrepreneur.† This is not the case in our model. We also do not obtain the result that the sequence of investments is always increasing.

Fifth, we construct two extensions to the main model. One is when both agents can repudiate the initial contract. We find that in this case there is some efficiency improvement in comparison with the main model. The other extension is when there are only costs of staged financing due to chopping of investment period. We discover that this extension is equivalent to the main model when there is a high enough outcome and no discounting. We use a graphical interpretation to illustrate the solution to this extension.

Appendix

Proof of Proposition 3.1

Before we proceed let us bring to mind all the equations we are going to use. The problem of the entrepreneur is

$$\beta^T R - \sum_{t=1}^{T} \beta^{t-1} I_t \longrightarrow \max_{T,\ \{I_t\}_{t=1}^{t=T}} \tag{3.11}$$

subject to (3.1), (3.2) and (3.10), that is

$$\sum_{t=1}^{T} I_t = K, \quad R = \int_0^K s(x)dx \quad \text{and} \quad \sum_{i=1}^{t} \beta^{i-t} I_i \le U_t, \quad 1 \le t \le T.$$

†In Appendix we present a condition on the rate of return function that is sufficient to ensure that the first of the results is satisfied.

The conditions of Proposition 3.1 are

$$U_t = L_t, \;\; 2 \leq t \leq T, \tag{3.12}$$

$$\sum_{i=1}^{t} \beta^{i-t} I_i = U_t, \;\; 2 \leq t \leq T, \tag{3.13}$$

$$U_1(T) \geq I_1(T) \text{ and } U_1(T-1) \leq I_1(T-1), \tag{3.14}$$

and

Minimal Cost Condition 2 *If there is only one investment path that satisfies conditions (3.12), (3.13) and (3.14), this path satisfies Minimal Cost Condition. If more than one investment path satisfies conditions (3.12), (3.13) and (3.14), there is a unique path that satisfies Minimal Cost Condition with the following pecking order property:*

- *of all the possibilities select the path with the lowest value of* I_1*;*
- *of all the possibilities that satisfy the previous item select the path with the lowest value of* I_2*;*
- *of all the possibilities that satisfy previous items select the path with the lowest value of* I_3*;*

and so on up until period T*.*

To prove the proposition, we firstly assume that T is as small as possible (condition (3.14) holds) and find which path is optimal in this case. We show that such a path has to satisfy conditions (3.12), (3.13) and the *MCC.* To prove that this path is optimal with respect to all feasible paths we show that it delivers higher utility than any path for $T' > T$.

(3.14) $\Longrightarrow$ (3.12)

First, we prove that from condition (3.14), condition (3.12) follows. That is for the minimal feasible T

$$L_t \geq S_t(T)/2 \quad \forall\ t = 2, 3, \dots, T.$$

This statement can be proved by contradiction. Assume that for optimal investment path $I_1, \dots, I_T$ for some $t \geq 2$ this property does not hold.[†] Let us construct some other shorter path with $T' = T - t + 1$, where $I'_1 := \Lambda_t$, $I'_2 := I_{t+1}, \dots, I'_{T-t+1} := I_T$. This path satisfies all the feasibility constraints because the original path satisfies them. Examine this issue in more detail.

The feasibility constraint (3.1) is satisfied by construction. Let us prove that the feasibility constraint (3.10) with respect to I'_1 holds, that is $I'_1 = I_1 + \dots + I_t \leq S'_1(T - t + 1)/2 = U'_1$, where $S'_1(T - t + 1)$ is a surplus to be bargained over in period one. On the one hand, it is straight forward to show that this surplus is the same as $S_t(T)$ for the initial path, that is $S'_1(T - t + 1) = S_t(T)$. On the other hand, from the assumption that condition (3.12) does not hold in period t and from the feasibility constraint (3.10) for I_t it follows that $S_t(T)/2 = U_t \geq I_t + \dots + I_1/\beta^{t-1}$, where the right-hand side is always not less than $I_1 + \dots + I_T$. The last two facts prove the first feasibility constraint for the new path. If $t = T$, there is only one feasibility constraint, and as we have seen it holds. In this case we can go straight to the next step. If $t < T$, there are some other feasibility constraints. Our goal is to prove them as well.

Now we prove that the feasibility constraint with respect to I'_2 holds. We modify inequality (3.10) for period $t + 1$ to get[†]

$$I'_2 = I_{t+1} \leq L_{t+1} - I_1/\beta^t - \dots - I_t/\beta \leq L_{t+1} - (I_1 + \dots + I_t)/\beta = L'_2 - I'_1/\beta.$$

[†]If there is a set of indexes where condition (3.12) does not hold then t is the largest index in this set.

[†]Note, that in period $t + 1$ condition (3.12) holds because by definition t is the largest among indexes for which that condition does not hold.

The same way we prove the feasibility constraint for any I'_i, where $2 \le i \le T - t + 1$

$$\begin{aligned} I'_i = I_{i+t-1} = L_{i+t-1} - I_1/\beta^{i+t-2} - \ldots - I_{i+t-2}/\beta < \\ < L_{i+t-1} - (I_1 + \ldots + I_t)/\beta^{i-1} - I_{t+1}/\beta^{i-2} - \ldots - I_{i+t-2}/\beta = \\ = L_{i+t-1} - I'_1/\beta^{i-1} - I'_2/\beta^{i-2} - \ldots - I'_{i-1}/\beta. \end{aligned}$$

Thus, the constructed $T - t + 1$ path is feasible. Let us show that it delivers higher utility than the original path. To do this we consider another T period path, that has zero investments during the first $t - 1$ periods, and then coincides with the $T - t + 1$ path, that is $I''_i = 0 \ \forall \ i = 1, \ldots, t-1$, $I''_t = I_1 + \ldots + I_t$, $I''_i = I_i \ \forall \ t+1 \le i \le T$. Now we show that the new T-period path $\{I''_i\}_{i=1}^{i=T}$ delivers higher utility than the original T-period path and lower utility than the $T - t + 1$ path, see Figure 3.9.

The first step is quite straightforward. Both paths generate the same return $\beta^T R$, while the original path has higher costs of delay because some of the investments were made earlier

$$\beta^T R - \sum_{t=1}^{T} \beta^{t-1} I_t \le \beta^T R - \sum_{t=1}^{T} \beta^{t-1} I''_t.$$

The second step is also not difficult. The new T-period path is actually $T - t + 1$-period path that starts after waiting for $t - 1$ periods. Because of costs of delay the utility from the constructed T-period path will be $1/\beta^{t-1}$ smaller than the utility from the $T - t + 1$-period path

$$\beta^T R - \sum_{t=1}^{T} \beta^{t-1} I''_t = \beta^{t-1}(\beta^{T-t+1} R - \sum_{t=1}^{T-t+1} \beta^{t-1} I'_t) \le \beta^{T-t+1} R - \sum_{t=1}^{T-t+1} \beta^{t-1} I'_t.$$

Thus, we have showed that the constructed $T - t + 1$-period path is both feasible and delivers higher utility than the optimal T-period path. We get a contradiction.

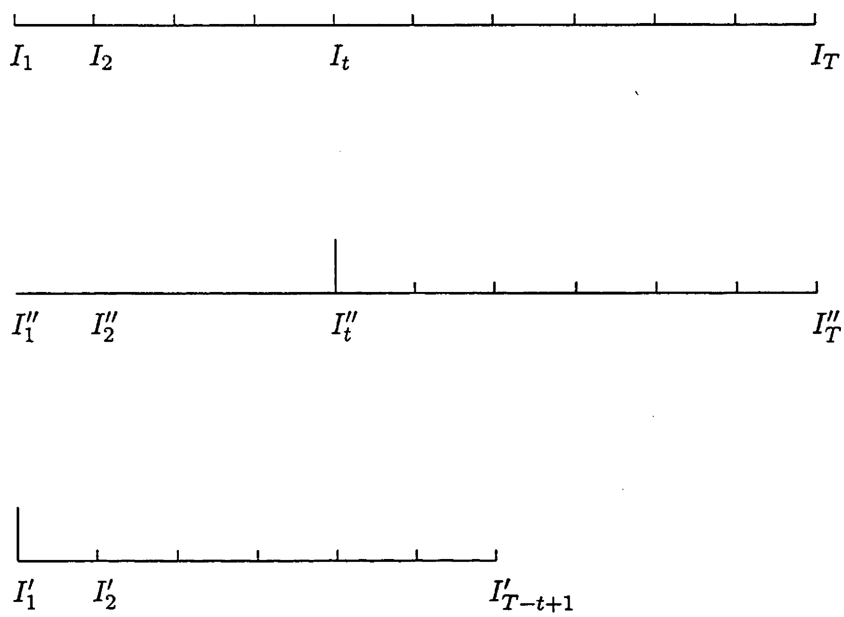

Figure 3.9: Investment paths $\{I_i\}_{i=1}^{i=T}$, $\{I''_i\}_{i=1}^{i=T}$ and $\{I'_i\}_{i=1}^{i=T-t+1}$

(3.14) and (3.12) $\Longrightarrow$ (3.13)

Second, we show that condition (3.13) also has to be satisfied when conditions (3.14) and (3.12) hold. Here we use the same approach as Neher (1999): we prove the statement by contradiction. Assume that for optimal investment path $I_1, \ldots, I_T$ for some $t \geq 2$, the following strict inequality holds $\sum_{i=1}^{t} \beta^{i-t} I_i < U_t$. To produce a contradiction we construct another investment path by moving ϵ investment from I_{t-1} to I_t and show that all the feasibility constraints hold and the new path is preferred by the entrepreneur.† Let us examine this statement in more detail.

With respect to constraints for periods after t this change is feasible because it only diminishes the costs of the previous investments. From condition (3.12)

† I_{t-1} has to be strictly positive, otherwise the original investment path is not optimal as $t-1$-th piece of investment can be removed.

the right-hand side of (3.10) is $U_i = L_i \ \forall \ i = 2, \ldots, T$. These liquidation values are the same after the change, while the left-hand sides of (3.10) diminish because ϵ investment was moved to a later period. Thus after the change the feasibility constraints for periods after t are satisfied.

The constraint for t is satisfied because ϵ is chosen to be small enough.

The constraints for periods before t are satisfied because they were satisfied before the change was made. Both the left-hand side and the right-hand side of (3.10) are exactly the same as before.

The change is beneficial for the entrepreneur, because it only diminishes the costs of the investment. Thus, we get a contradiction because the original investment path was optimal.

(3.12), (3.13) and (3.14) $\Longrightarrow$ *MCC*

Consider investment path $I_1, \ldots, I_T$ that satisfies conditions (3.12), (3.13), (3.14) and the *MCC* and some other investment path $I'_1, \ldots, I'_T$ that satisfies only (3.12), (3.13) and (3.14), but does not satisfy the *MCC*. Let us prove that $\Lambda_t \leq \Lambda'_t \ \forall \ t = 1, 2, \ldots, T-1$.

The inequality for $t = 1$ follows straight away from condition *MCC*: $\Lambda_1 = I_1$ and I_1 is the minimum.

The inequality for $t = 2, \ldots, T-1$ can be proved by contradiction. Imagine $\Lambda_t > \Lambda'_t$ for some $t = 2, \ldots, T-1$. Let us construct some other path $\{I''_t\}_{t=1}^{t=T}$ that has $\Lambda''_t = \min[\Lambda_t, \Lambda'_t] \ \forall \ t = 2, \ldots, T$. On the one hand, it is straight forward to show that this path satisfies all the feasibility constraints because both original paths satisfy them.† On the other hand, this path delivers higher utility than any of $\{I_t\}_{t=1}^{t=T}$ and $\{I'_t\}_{t=1}^{t=T}$ investment paths because it has smaller costs.† Thus, we show that condition (3.13) is not necessary for an optimal investment path, which

†It is always possible to move from the minimum for some period i to either the minimum or the maximum for the next period $i+1$, because one of the original paths can do it and the new path has smaller costs. It means that it is always possible to move to the minimum because the constraint for period $i+1$ allows the investor to invest less.

†The formal proof of this statement can be seen in the next paragraph.

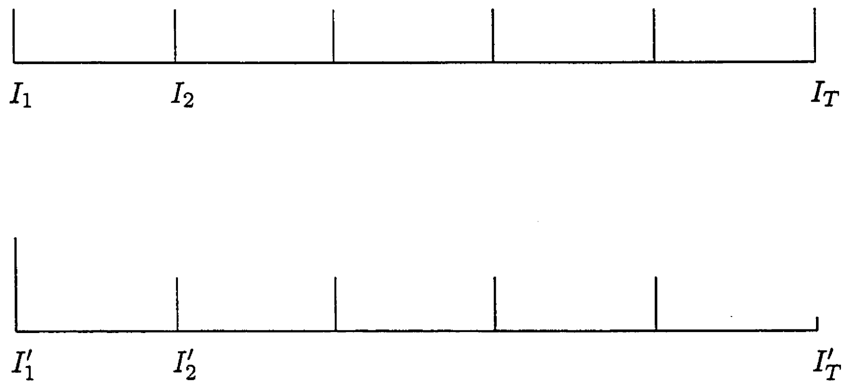

Figure 3.10: Investment paths $\{I_t\}_{t=1}^{t=T}$ and $\{I'_t\}_{t=1}^{t=T}$

contradicts to previous sections of the proof.†

Next step is to show that path $\{I_t\}_{t=1}^{t=T}$ that has the property $\Lambda_t \leq \Lambda'_t \quad \forall \quad t = 1, 2, \ldots, T-1$ delivers higher utility than $\{I'_t\}_{t=1}^{t=T}$. Note, that both investment paths have the same return $\beta^T R$. Let us show that the costs for the investment path $\{I_t\}_{t=1}^{t=T}$ are smaller.

Let us firstly compare costs of I_T investment for both paths. Note, that $I_T \geq I'_T$ because $\sum_{t=1}^{t=T} I_t = \sum_{t=1}^{t=T} I'_t = K$ and $\Lambda_{T-1} \leq \Lambda'_{T-1}$. It means that for $\{I_t\}_{t=1}^{t=T}$ path the whole I_T was invested in the last period, while for $\{I'_t\}_{t=1}^{t=T}$ path only some part of I_T was invested in the last period and the rest was invested in period $T-1$. So, I_T has higher costs for path $\{I'_t\}_{t=1}^{t=T}$. Next, we compare costs of $I_{T-1}, \ldots, I_1$ investments for both paths and using the same approach get similar results. Thus, the costs of $\{I_t\}_{t=1}^{t=T}$ investment path are smaller, which means that it delivers higher utility than the $\{I'_t\}_{t=1}^{t=T}$ investment path, and consequently we prove that condition MCC is necessary.

To summarize, we have proved that for the minimal feasible T the optimal path

†If investment paths $\{I_t\}_{t=1}^{t=T}$ and $\{I''_t\}_{t=1}^{t=T}$ are different then at least in some period t $I''_t < I_t$, which means that condition (3.13) is not satisfied on investment path $\{I''_t\}_{t=1}^{t=T}$, because by construction it is satisfied on $\{I_t\}_{t=1}^{t=T}$.

satisfies conditions (3.12), (3.13) and the *MCC*. Now the last part of the proof, we show that this path, that is the path satisfying conditions (3.12)-(3.14) and the *MCC*, yields to the entrepreneur a higher utility than any feasible path for some larger T.

(3.12)-(3.14) and *MCC* gives higher utility than any $T' > T$ path

Consider optimal investment path for T, namely $I_1, \ldots, I_T$ and any path for $T+i$, namely $I'_1, \ldots, I'_{T+i}$. Note that the path for $T+i$ during the first $i+t$ periods invests not less than Λ_t. Let us consider this statement in more detail.

Firstly, we assume that $\Lambda'_{i+1} < I_1$. We construct another T-period path $\Lambda''_t = \min[\Lambda_t, \Lambda'_{i+t}]$, $\forall\ t = 1, \ldots, T$. Let us prove that this path is feasible. The feasibility for $t = 2, \ldots, T$ can be proved using the same approach as in the previous subsection. The costs for the new path are smaller than in any of two original paths, and that allows us to move from the minimum to the minimum. The feasibility for $t = 1$ is a bit more tricky. To prove it let us rewrite the first feasibility constraint for $I_1, \ldots, I_T$ path in the following way

$$I_1 \leq \beta^T (R - I_T/\beta - \ldots - I_2/\beta^{T-1} - I_1/\beta^T).$$

From this inequality it follows that the first feasibility constraint for the new path is satisfied because $I''_1 < I_1$ and the costs for the new path are smaller. Thus, the new path is feasible and it delivers higher utility than the original T-period path, which is optimal among T-period paths. This is a contradiction.

To prove the original statement for any $2 \leq t \leq T$ we again use the same T-period path $\Lambda''_t = \min[\Lambda_t, \Lambda'_{i+t}]$, $\forall\ t = 1, \ldots, T$. We show that the new path is feasible and delivers higher utility than the original T-period path. We get a contradiction and conclude the proof of the original statement.

The property $\Lambda_t \leq \Lambda'_{t+i}$ ensures that the investment path $I_1, \ldots, I_T$ delivers higher utility than $I'_1, \ldots, I'_{T+i}$.[†] This observation concludes the last part of the

[†] For the prove of a very similar statement see the previous subsection.

proof.

Note, that the investment path satisfying conditions (3.12)-(3.14) and the MCC is unique by construction.

Proofs of results

Result 3.2

To prove Result 3.2 we use Assumption (3.16) for period T, that is

$$S_T \geq L_T, \tag{3.16}$$

where from equation (3.5)

$$S_T = \beta R,$$

and from equation (3.15)

$$L_T = \sum_{t=1}^{t=T} \beta^{t-T} I_t.$$

We multiply inequality (3.16) by β^{T-1} and get

$$\beta^T R - \sum_{t=1}^{T} \beta^{t-1} I_t \geq 0,$$

which means that the entrepreneur's utility (3.7) is always non-negative on the optimal investment path once Assumption (3.16) is satisfied.

Result 3.3

Now we prove that Assumption (3.16) is always satisfied on the optimal investment path. Firstly, note that this assumption holds for $t = 1$. S_1 is positive because the project is profitable and $L_1 = 0$ by construction, that is

$$S_1 > 0 = L_1.$$

Next, prove it for $t = 2$. This is easy to do with help of modified equations (3.5), (3.15) and (3.14), that is

$$S_2 = S_1/\beta + I_2, \;\; L_2 = I_1/\beta + I_2 \text{ and } I_1 \leq S_1/2 \implies S_2 \geq L_2.$$

Now, let us prove Assumption (3.16) $\forall\ t = 3, \ldots, T$. Because $S_2 \geq L_2$ from modified equations (3.5) and (3.15)

$$S_t = S_{t-1}/\beta + I_t, \;\; L_t = L_{t-1}/\beta + I_t \implies S_t \geq L_t \;\forall\; t = 3, \ldots, T.$$

Result 3.4

Here we prove that the optimal investment path derived by the algorithm is better than any other repudiation-proof path for which Assumption (3.16) is not satisfied.

Let us consider an investment path that has

$$S_t < L_t$$

for some $2 \leq t \leq T$, and let us prove that in that period condition (3.10) is satisfied. That is

$$\sum_{i=1}^{t} \beta^{i-t} I_i \leq L_t. \tag{3.10}$$

To prove this inequality we use the fact that the project is profitable, that is

$$\beta^T R - \sum_{i=1}^{i=T} \beta^{i-1} I_i > 0.$$

We divide it by β^{t-1} and using equation (3.5) derive

$$S_t > \sum_{i=1}^{t} \beta^{i-t} I_i.$$

Combining this inequality with the assumption that $L_t > S_t$, we prove condition (3.10) for period t. It means that in all periods where $L_t > S_t$ condition (3.10) is satisfied. In other periods it is also satisfied because the investment path is repudiation-proof. This means that the investment path is feasible for optimization problem (3.11). Once it is feasible it is worse than the optimal investment path constructed by Algorithm 1.

Result 3.5

Here we prove that the optimal strategy for the entrepreneur is to invest all his wealth at the very beginning. The proof consists of two parts. In the first part we prove that it is optimal to invest all the wealth at the very beginning rather than in some later time. In the second part we show that it is optimal to invest the wealth rather than only use it as a collateral.

Let $\{I_t\}_{t=0}^{t=T}$ be optimal investment path with the entrepreneur's wealth being invested at the beginning of the second period as a part of I_2 (we call it an old path). There exist two cases: $I_1 \geq \omega$ and $I_1 < \omega$. For the first case let us show that $\{I_t\}_{t=0}^{t=T}$ path is feasible when the entrepreneur's wealth is invested at the beginning of the first period as a part of I_1 (we call it a new path). Both paths produce the same return for the entrepreneur. Let us use feasibility of the old path to prove feasibility of the new path. Condition (3.15) for the old path for periods $t > 2$ is exactly the same as for the new path. Changes appear in the first two periods only and do not affect the later periods. Thus, we only need to examine whether conditions for the first two periods are satisfied. First, consider condition (3.14) for the first period. The investor's part in the first period for the new path is $I_1 - \omega$, which is less than his part for the old path, I_1. It means that condition (3.14) for the new path holds once the similar condition for the old path holds. Now we prove that condition (3.15) for the second period holds. The analog of condition (3.15) for the old path for $t = 2$ is $L_2 + \omega = I_2 + I_1/\beta$. On the other hand, the feasibility constraint for the new path is $L_2 \geq I_2 + (I_1 - \omega)/\beta$ and it can be easily derived from the previous equality. Thus, the new path is feasible, but not optimal (condition (3.15) is not satisfied for the second period).

As the next step consider the second case with $I_1 < \omega$. Using the similar technique as in previous paragraphs one can prove that it is optimal for the entrepreneur to invest in the first period at least I_1 by himself. Let us prove that it is optimal to invest the whole ω in the first period. Before we begin our proof let us find an answer to the following question. If the entrepreneur does invest only I_1 in the first period what is his best strategy afterwards? From the previous paragraph we know that he would invest the rest in the second period, if the second-period investment is not large enough then what is left goes to the third period and so on. Now, we compare the case when the entrepreneur invests the whole wealth at the first period and the case when he invests it gradually. To realize that the first case is better note that the entrepreneur bears no costs on his investment in this set-up, while there are benefits from investing earlier (the project can be finished earlier). This observation concludes the second step of the proof.

If the entrepreneur's wealth is invested in some period $t > 2$, then using the same technique one can prove that the corresponding optimal investment path is dominated by the optimal path with the entrepreneur's wealth invested in the first period.

Now let us show that it is optimal to invest all the wealth rather than use it as a collateral. The case with collateral is even worse than investing the wealth in period T. If it is not invested in period T then the costs of it are ω/β, while the benefits are ω. This observation concludes the proof.

Result 3.7

In the time interval between this period's 'Investment' and the next period's 'Investment', the investor has exactly the same outcomes of repudiation. As there is discounting he has an incentive to repudiate only at the beginning of this time interval, that is straight after he supplies the next piece of investment. He does not repudiate at any other moments in given period t.

With respect to the entrepreneur, everything is exactly the same as in the main model. It is optimal to repudiate straight after the investor gets another piece of investment sunk.

Proofs of other results

Proof of results in example 1

For a some large number T let us find the optimal investment path using the following algorithm, which is a bit different from algorithm 1. Firstly, let us use condition (3.14)

$$U_1 = S_1/2 = (R - \sum_{t=2}^{t=T} I_t)/2 = (2(K-c) - (K - I_1))/2 \geq I_1,$$

that means

$$I_1 \leq K - 2c.$$

Now use condition (3.15) for $t = 2$ to get

$$I_2 = L_1 - I_1 = 2(I_1 - c) - I_1 = I_1 - 2c \leq K - 4c.$$

Next, let us prove that $I_t = 2I_{t-1} \; \forall \; t \in [3, T]$. Use condition (3.15) for periods $t-1$ and $t \quad \forall \; t \in [3, T]$

$$L_t = I_1 + \ldots + I_{t+1} \text{ and } L_{t-1} = I_1 + \ldots + I_t.$$

From equation (3.3) that define L_t and L_{t-1} it follows

$$L_t = L_{t-1} + 2I_t.$$

Combining these facts together we get $I_t = 2I_{t-1} \; \forall \; t \in [3, T]$. The proof is complete.

Decreasing sequence of investments

Let us consider the following example. The first dollar of investment returns 2 dollars, the second dollar of investment returns 0.6 dollars and the last 0.6 dollars return 1.2 dollars, see Figure 3.11.

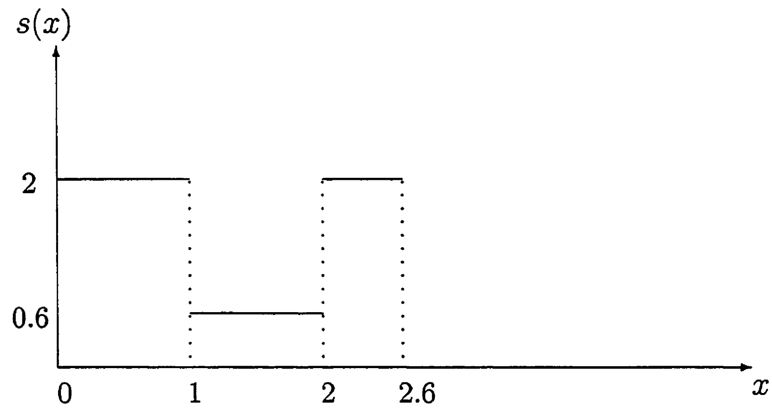

Figure 3.11: Rate of return $s(x)$

Let us show that investment path $I_1 = \$1$, $I_2 = \$1$ and $I_3 = \$0.6$ is optimal. Firstly, the project is not financed in one period because $S_1(1)/2 = R/2 = \int_0^{2.6} s(x)dx/2 = \$1.9 < K = \$2.6$.

Secondly, the project is not financed in two periods. Investments $I_1 = \$2$ and $I_2 = \$0.6$ satisfy condition (3.15), $L_2 = \int_0^{I_1} s(x)dx = \$2.6 = I_1 + I_2 = \$2.6$. Let us examine condition (3.14): $S_1(2)/2 = (R - I_2)/2 = \$1 < I_1 = \$2$. Condition (3.14) is not satisfied, which means that the project is not financed in two periods.

Now we take three-period investment path $I_1 = \$1$, $I_2 = \$1$, $I_3 = \$0.6$ and show that it is feasible. Condition (3.15) is satisfied, $L_2 = \int_0^{I_1} s(x)dx = \$2 = I_1 + I_2 = \$2$ and $L_3 = \int_0^{I_1+I_2} s(x)dx = \$2.6 = I_1 + I_2 + I_3 = \$2.6$. Condition (3.14) is also satisfied, $S_1(3)/2 = (R - I_2 - I_3)/2 = \$1 = I_1 = \$1$.

Thus, the three-period investment path is optimal. One can see that this path is decreasing.

Conjectures to extensions of Neher (1999)

In this section we generalize comparative statics and extensions of Neher(1999). The following propositions (3.3), (3.4) and (3.5), are modified versions of propositions 4, 4A and 4B of Neher(1999), that hold for the case of a variable s.

Comparative statics with respect to β and $s(x)$

Proposition 3.3 *For any given project consider the optimal investment path constructed using the algorithm 1. As β or $s(x)$ increases, the path continues to satisfy all the feasibility constraints.*†

Proof. If β increases then the left-hand sides of feasibility constraints (3.10) decrease and the right-hand sides of feasibility constraints (3.10) do not decrease (liquidation values stay the same, while the surpluses increase). It means that after the change in β the old optimal investment path is still feasible.

The increase in $s(x)$ only affects the liquidation values and the surpluses. Both constitute the right-hand sides of constraints (3.10) and both increase. Again the old optimal investment path is still feasible.

In other words, an increase in β decreases the costs associated with delay, while an increase in $s(x)$ increases the profitability of the project. Both of the changes have only positive effects and can only make the entrepreneur better off. The same investment path has to be feasible. □

Let us define

$$L(s(x)) := \int_0^x s(x)dx. \tag{3.26}$$

An interesting corollary to Proposition (3.3) is

Corollary 3.3 *If $L(s(\cdot))$ is first order stochastically dominated by $L(s'(\cdot))$ then the optimal investment path for $s'(\cdot)$ is feasible with $s(\cdot)$.*

† *By increase in $s(x)$ we mean increase in s for some x, where s for all other x is not less than previously.*

Proof. Similar to the proof of the proposition. If the liquidation values for s are always higher than the liquidation values for s' on the same investment path then the path, that is feasible with s', will be feasible with s. □

Change in tangibility of the physical assets

To construct Proposition 3.4 let us consider the following extension to the model. Now, let the last piece invested I_t, not be completely sunk. It means that if I_t has not had the whole period with the entrepreneur, it is possible to trade I_t for αI_t where $0 \leq \alpha \leq 1$. If I_t has had the whole period with the entrepreneur then it has the same value as in the main model. So the liquidation value of the project for period t is

$$L_t := \int_{x=0}^{\Lambda_{t-1}} s(x)dx + \alpha \int_{\Lambda_{t-1}}^{\Lambda_t} s(x)dx. \tag{3.27}$$

This little change to the model does not change the algorithm of finding the solution. The result of the Proposition 3.1 is valid. The following comparative statics with respect to this extension holds.

Proposition 3.4 *For any given project consider the investment path constructed using the algorithm 1 with L_t defined by equation (3.27). As α increases the path continues to satisfy all the feasibility constraints.*

Proof. An increase in α only increases the liquidation values that stay in the right-hand side of feasibility constraints (3.10). After that change, the right-hand sides can only increase and the feasibility constraints will be satisfied. □

Growth of physical assets in outside value

Another possible change to the main model is to let the physical assets that have had at least one period with the entrepreneur grow in outside value at rate γ. It means that the liquidation value of the project for period $t+1$ is

$$L_{t+1} = \gamma L_t + \int_{\Lambda_{t-1}}^{\Lambda_t} s(x)dx. \tag{3.28}$$

This change in liquidation value also does not alter the algorithm of finding the optimal investment path. The following comparative statics with respect to this extension holds.

Proposition 3.5 *For any given project consider the investment path constructed using the algorithm 1 with L_t defined by equation (3.28). As γ increases the path continues to satisfy all the feasibility constraints.*

Proof. The proof of this proposition is very similar to the proof of the previous proposition. Increase in γ only increases the liquidation values that stay in the right-hand side of feasibility constraints (3.10). After that change, the right-hand sides can only increase and the feasibility constraints will be satisfied. □

Sufficient condition for one of the Neher's results

In this section we derive sufficient conditions for the following result of Neher to be satisfied: as $\beta \to 1$ all profitable projects become financeable.

From Neher (1999) we know that once the rate of return function is constant the result mentioned above holds. Let us introduce such changes to the rate of return function that this property does not change. Really, let us assume that the rate of return function is monotonically decreasing and compare this situation with the situation when the rate of return function is constant and equal to the average rate of return for the variable case.[†] It is clear that the optimal investment path for the constant rate of return case is feasible with the variable rate of return.[†] Once it is feasible the result of Neher, that is all the profitable projects are financed when $\beta \to 1$, holds.

Result 3.8 *As $\beta \to 1$ all profitable projects with monotonically decreasing rates of returns become financeable.*

Proof. The proof follows from the discussion above and is omitted. □

[†]It means that the final returns are the same for both cases.

[†]The only change that arises with the shift from the constant rate of return case to the variable rate of return case is that liquidation values increase (the right-hand sides of feasibility constraints (3.10) increase). This change does not break the feasibility of the investment path.

CHAPTER 4

Hold-up and sequential specific investments

4.1 Introduction

Many projects prior to their commencement are nebulous and difficult to describe. For example, research and development projects often have vague objectives and speculative or uncertain outcomes; start-up firms are often based around intangible ideas. With joint projects this makes it difficult to write a complete contract specifying the tasks of each party and the desired outcome (see for example Hart 1995, pp. 1-5). Grout (1984) and Hart (1995), amongst others, showed that parties may not make efficient specific investments when contracts are incomplete.[†] These models typically have the following structure: trading parties make their investments that are sunk and, at least partially, specific; after these investments are made contracting on some relevant variable becomes possible; at this point the parties renegotiate and trade occurs according to the renegotiated contract. If, because of renegotiation, a party does not receive the full marginal return from their effort, investment will be inefficient.[†]

Other authors have examined how sequencing or staggering investment can help alleviate the hold-up problem. For example, Neher (1999) considered staged financing of a project when an entrepreneur is unable to commit not to renege on their

[†]Also see Grossman and Hart (1986) and Hart and Moore (1988).

[†]It has also been noted that the level of general investments can be effected in the presence of incomplete contracts: Malcomson (1997) noted that hold-up of general investment can occur when there are turnover costs.

contract with the financier. When the project is financed in stages, as the project matures, the alienable (contractible) element of the project, manifested in the accumulated physical assets, provides a better bargaining position during renegotiation for the financier in the event of default. As a consequence, the entrepreneur has less incentive to renege. De Fraja (1999) considered the Stackelberg-type sequencing of investments in the presence of hold-up. De Fraja's solution to the hold-up problem required the first party to make a general investment, then make a take-it-or-leave-it offer to the other player that included the first party paying for the specific investment.[†] Given that this contract makes the first party the residual claimant she will invest efficiently. Admati and Perry (1991) showed two parties can overcome the free-rider problem by financing a public good in stages.

The model presented here develops a simple framework to contrast the simultaneous and staged (sequential) investment regimes. The essence of the model is that staging the project allows some investment to be made after the point in time when a contract can be written. Here, the resolution of the incompleteness is facilitated by the completion of some aspect of the project. Similarly, in Grossman and Hart (1986) contracting became possible after the two parties made their initial investment. Further, Neher (1999) made the point that contracting becomes progressively more feasible as human capital invested in the project is converted into physical assets.

The basic structure of the model is as follows. Two parties are required to invest in order to complete a project. Two distinct alternatives are available to the parties. First, they can invest simultaneously at the start of the game. If they do so, both invest prior to when complete contracting is possible. After both investments are sunk the parties renegotiate and the payoffs are realized.[†] Alternatively, one party can invest first while the other party waits. This first investment allows the project to take shape: as a result, contracting on the second investment becomes possible.

[†]Although the investment may be industry-specific, it is not relationship-specific in the traditional sense. See Malcomson (1997).

[†]This regime is equivalent to the incomplete contract models of Grossman and Hart (1986) and Hart (1995).

At this stage, the parties will renegotiate and write a contract specifying the second party's investment. The final stage of investment will then occur, completing the project and allowing the parties to receive their payoffs.

As an example to motivate the model, a popular music recording can either be completed simultaneously or sequentially.[†] An artist can write the songs, record them and go to the record company who then makes their investment in promotion, printing and distribution. Alternatively, both parties can make investments simultaneously in choosing the songs, recording, promotion and the like. These investments by the artist and the record company can be specific if both parties have an overarching contract to deal with one another. Further, this contract is likely to be incomplete in that only after the recorded product is finished can the parties decide on important issues, such as which songs should be released as a single, etc.[†]

Several important results arise from this simultaneous versus sequential investment model. First, the sequential regime can create trading possibilities that may not be feasible if the parties have to invest simultaneously. For example, the second player will not be willing to invest simultaneously if it leaves them with a negative net return. On the other hand, the sequential investment regime gives this player the opportunity to delay their investment until when contracts are complete. This may be sufficient incentive to encourage the seller to invest. This result is similar to the results of other authors, for example Neher (1999) and Admati and Perry (1991), albeit in a different context.

Second, we show that the possibility of investing sequentially does not always improve welfare. As it turns out, flexibility in the timing of investment can act as an additional form of hold-up. For want of a better expression we call this kind of hold-up, 'follow-up'. This occurs when both parties should invest simultaneously at the start of the project in order to maximize surplus, but there is an incentive for one party to wait until after the other player has sunk their effort before they

[†]Also see Hart (1995) for an example of sequential investment with incomplete contracts by a building contractor and a purchasing party. (pp. 1-5)

[†]For a detailed description of the music industry see Fink (1996).

follow-up with their own investment.[†] Consider the case when technology requires that one particular party must invest at the commencement of a project but that the other party can invest either at the same time or wait. The first party will anticipate that the second party will delay their investment - opt for the sequential regime - if it suits them.

Third, as discussed above, the second player acting in self-interest may have the incentive to opt for the regime that does not maximize total welfare. The burden of this opportunism is typically borne by the other player. However, if such opportunism drives the first player's return below his no-trade payoff, this additional form of hold-up will prevent a potential surplus-enhancing project from proceeding. In this case, the second player also bears some of the cost from the reduction in total surplus - the second player is disadvantaged by her inability to commit to a particular timing schedule of investment.

Fourth, the decision over the timing of investment can be seen as a choice over the completeness of contracts: if parties opt for simultaneous investment they are opting for a more incomplete contract than possible (with the sequential regime). As a result, the choice concerning the completeness of contract is endogenous. The advantage of a (more) complete contract with sequential investment is that hold-up of the follower is avoided. The cost of a complete contract is that it diminishes the first party's incentive to invest and increase the costs of delay. The second party will opt for simultaneous investment - that is, they will opt for an incomplete contract - when their gain from the increase in total surplus outweighs the additional bargaining power they receive from avoiding hold-up.[†]

Finally, interesting dynamics can arise out of this investment game when both parties want to be a follower rather than the leader. If there are just two potential investment periods (and the opportunity to invest disappears after the second period) the parties find themselves in a prisoners' dilemma. If the potential investment horizon is continually extended to three periods, four periods and so on, eventu-

[†]'Follow-up' can occur in addition to the regular hold-up of investment.

[†]In their study of the Coase theorem, Pitchford and Snyder (1999) also developed a model that generates endogenous incomplete contracts.

ally the benefit from not investing (waiting) will diminish sufficiently so that the players will find themselves in a coordination game. (The players will mix between investing immediately and waiting.) If the horizon is extended further from this point, with certain parameter values it is possible that the players will again return to a prisoners' dilemma game. This arises because the payoff in the coordination game (say in period K) alters the expected return from waiting in the game with the longer horizon (say a game of $K + 1$ periods). It is possible that the optimal strategies switch between a prisoners' dilemma game and a coordination game as the potential horizon is extended. The equilibria in the potential infinite horizon game are also examined.

4.2 The model

There is a potentially profitable relationship between two parties that, for convenience, we label as a buyer and a seller. Specifically, if the buyer and seller invest I_1 and I_2 respectively the two parties share surplus R. The exact relationship between the investments and surplus is discussed below.

The timing of investment is the focus of this chapter. Two alternatives are considered. First, both players invest simultaneously at time $t = 1$, as shown in Figure 4.1. At this stage, contracting on either investment is not possible; consequently renegotiation (or contracting) will occur after both investments are sunk.[†] Definition 4.1 reiterates this discussion.

Definition 4.1 *Simultaneous investment occurs when both parties invest at the same time, prior to renegotiation.*

Figure 4.2 outlines the timing of the alternative investment regime. In this regime the buyer invests I_1 at time $t = 1$ prior to when contracting is possible. However, this investment makes contracting possible, so having observed I_1 the two parties renegotiate and contract on I_2. It is only at this stage that the seller makes

[†]The renegotiation process is discussed below.

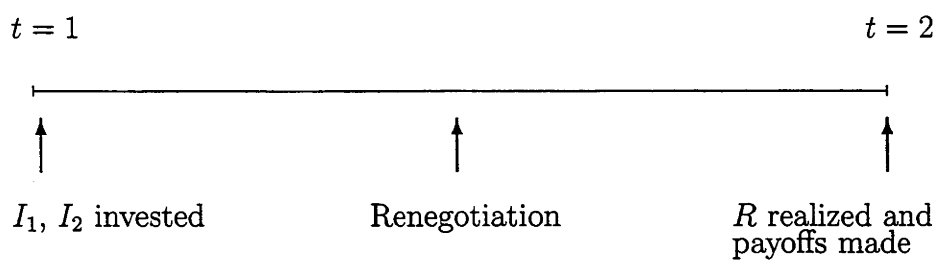

Figure 4.1: Simultaneous investment

her investment I_2. This occurs at time $t = 2$. After both investments have been made, surplus is realized and the payoffs to each party are made. Definition 4.2 defines sequential investment.

Definition 4.2 *Sequential investment occurs when one party (the buyer) invests at time $t = 1$, while the other party (the seller) waits and invests at time $t = 2$.*

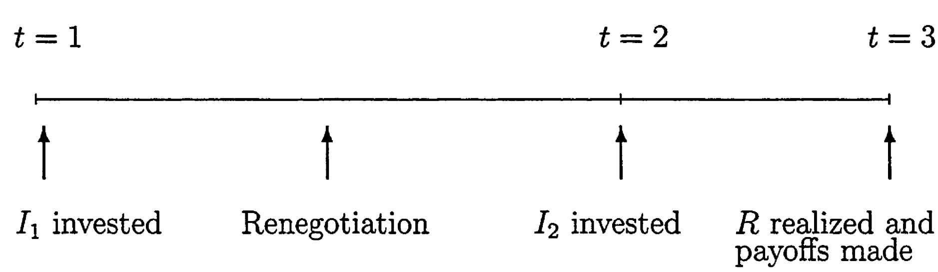

Figure 4.2: Sequential investment

As noted above, the investments of the buyer and seller (I_1 and I_2) combine together to generate surplus R. The investments of both parties are sunk and completely specific to the relationship, in that they are worth zero outside the relationship. R is only available at the completion of the project. For simplicity we assume the buyer and the seller can make discrete investments of $I_1 = \{0, f_1\}$ and $I_2 = \{0, f_2\}$ respectively.† The surplus generated will be equal to R if both f_1 and f_2 are invested, and zero otherwise. The outside options of both players are normalized to zero. Further, trade between the buyer and seller is efficient; that is, $\delta^2 R - \delta f_2 - f_1 > 0$, where the discount factor δ is discussed below.† Assumption 4.1 summarizes this discussion.

Assumption 4.1 *The buyer can make investment $I_1 = \{0, f_1\}$ and the seller can make investment $I_2 = \{0, f_2\}$. If $I_1 = f_1$ and $I_2 = f_2$ surplus R is available to the two parties. The outside options of both players are normalized to zero and $\delta^2 R - \delta f_2 - f_1 > 0$.*

Although there is complete and symmetric information between the trading parties, the investments are unverifiable ex ante. However, as discussed above, once the buyer's investment has been sunk the project becomes tangible allowing subsequent investment to be verifiable. This can arise when the required tasks of the second party become evident after the project is underway. The buyer's investment, I_1, could also be thought of as an investment in writing a contract, or blueprint, for the desired trade. In this context the parties can opt to invest without a complete contract (simultaneous investment) or to opt for a (more) complete contract (the sequential regime).†

Unlike investment, the surplus generated by the project is always unverifiable. This prevents the parties writing surplus sharing agreements. Further to this, prior to the commencement of the project the parties are unable to write a fixed price

†In the discussion here, it is assumed that the level of investment by each player is discrete and, hence, fixed if they decide to invest. In the next chapter we explore the timing of investment and the potential for follow-up when investments are continuous.

†This assumption means that trade is efficient with both simultaneous and sequential investment as it follows from $\delta^2 R - \delta f_2 - f_1 > 0$ (the relevant condition for when investment is sequential) that $\delta R - f_2 - f_1 > 0$ (the relevant condition for simultaneous investment).

†Note, the idea that the buyer invests effort into writing a contract does not rule out the possibility that this blueprint is specific to the parties.

contract, as suggested by MacLeod and Malcomson (1993). These points are summarized in Assumption 4.2 and 4.3 and Remark 4.1.

Assumption 4.2 *Prior to the investment of I_1 both I_1 and I_2 are unverifiable, and hence non-contractible.*

Assumption 4.3 *Surplus $R(I_1, I_2)$ is unverifiable.*

Remark 4.1 *Surplus sharing agreements are not feasible.*

As in Hart and Moore (1988) and MacLeod and Malcomson (1993), the two parties cannot vertically integrate to overcome their hold-up problem. This could be due to specialization, for example. If the parties could vertically integrate, as noted by Williamson (1983), they could overcome hold-up and investment would be efficient. Assumption 4.4 outlines this point.

Assumption 4.4 *The two parties cannot vertically integrate.*

Further, both the parties discount future returns and costs with a constant discount factor $\delta \in (0, 1]$ per period, as stated in Assumption 4.5.

Assumption 4.5 *Both parties discount future costs and returns with the discount factor δ per period, where $\delta \in (0, 1]$.*

With the simultaneous regime, the returns accrue at $t = 2$, and are thus are discounted by δ. The sequential regime lengthens the entire investment process: an investment made after renegotiation at time $t = 2$ is discounted by δ while the returns are discounted by δ^2 as they accrue at time $t = 3$. The discount factor is included in the model on that basis that investment can take real time to complete. Moreover, for simplicity, each investment is assumed to take the same amount of time. Note, however, that the results presented in this chapter do not rely on the inclusion of an additional period with the sequential regime. The situation when there is no additional discounting with the sequential regime is equivalent to when $\delta = 1$.

When the parties renegotiate they must decide how to split the available surplus. For simplicity it is assumed that both parties have equal bargaining power. With zero outside options it means that each party receives one-half of the available surplus.†

Assumption 4.6 *Both parties have equal bargaining power and receive one-half of the available surplus.*

4.3 Follow-up and the timing of investment

First consider the outcome when the parties invest simultaneously. After investing f_1 and f_2, the parties will renegotiate over surplus R. As noted above, the parties will distribute surplus equally. The returns to the buyer and seller respectively are:

$$\frac{1}{2}\delta R - f_1; \tag{4.1}$$

and

$$\frac{1}{2}\delta R - f_2. \tag{4.2}$$

When only the simultaneous investment regime is available the buyer will anticipate a return of $\frac{1}{2}\delta R - f_1$ from within the relationship. Consequently, the buyer will opt into the investment relationship provided

$$\frac{1}{2}\delta R - f_1 \geq 0. \tag{4.3}$$

The buyer will opt not to enter the relationship if

$$\frac{1}{2}\delta R - f_1 < 0. \tag{4.4}$$

†The relative bargaining strengths of the parties are encapsulated in Chiu (1998) by α and $(1-\alpha)$, so that the first party receives α of the surplus. Here $\alpha = \frac{1}{2}$. These bargaining strengths depend on exogenous characteristics of the negotiating parties, for example their rate of time preference or their expectation of a breakdown in bargaining. As usual, sunk costs do not affect the distribution of surplus at renegotiation.

This is an example of the standard hold-up problem that arises with incomplete contracts. If contracting were complete, given overall surplus is increased within the specific relationship, the parties could contract on f_1 and ensure that the buyer receive surplus at least as great as 0. The same reasoning applies to the seller. If $\frac{1}{2}\delta R - f_2 \geq 0$ the seller will opt into the relationship. Conversely, if $\frac{1}{2}\delta R - f_2 < 0$ the seller will anticipate the hold-up problem and opt not to invest, reducing total surplus.†

Now consider when the parties can only invest sequentially. In this case the two parties will renegotiate after the buyer has sunk his investment, but prior to the seller investing f_2. If both parties invest in the relationship, the return of the buyer and seller, valued at $t = 1$, will be:

$$\frac{1}{2}(\delta^2 R - \delta f_2) - f_1; \tag{4.5}$$

and

$$\frac{1}{2}(\delta^2 R - \delta f_2). \tag{4.6}$$

The important element here is the treatment of the buyer and the seller in the renegotiation process. As the buyer has sunk their investment, f_1 does not affect the distribution of surplus. The seller, on the other hand, has not made her investment. Her investment f_2, as a consequence, is considered as part of net surplus the parties bargain over. In this sense, the seller avoids being held-up with sequential investment.

At this point we turn our attention to the situation when both regimes are possible. As noted in the literature, having the option of sequential investment can improve welfare. To see this consider the case when the buyer's outside option is never binding ($\frac{1}{2}(\delta^2 R - \delta f_2) - f_1 > 0$): this ensures that the buyer will opt into the relationship regardless as to whether investments are simultaneous or sequential. Further, assume $\frac{1}{2}(\delta^2 R - \delta f_2) > 0 > \frac{1}{2}\delta R - f_2$. As the seller's no trade option

†Up-front compensation will have limited success overcoming the hold-up problem, as fixed payments do not affect marginal incentives.

exceeds her return if investments are simultaneous ($\frac{1}{2}\delta R - f_2 < 0$) she would not enter the relationship if investments could only be made simultaneously. However the sequential regime may create an environment that helps facilitate trade between the parties. The seller will receive a payoff of $\frac{1}{2}(\delta^2 R - \delta f_2)$, valued at time $t = 1$, as the parties renegotiate after the buyer has invested but before the seller has done so. As noted above, this allows the seller to avoid being held-up: the extra surplus afforded the seller with sequential investment encourages her to invest where she would not otherwise done so. The allows trade to occur that would not be feasible with only simultaneous system available. Proposition 4.1 summarizes the discussion above.

Proposition 4.1 *When $\frac{1}{2}(\delta^2 R - \delta f_2) - f_1 > 0$ and $\frac{1}{2}(\delta^2 R - \delta f_2) > 0 > \frac{1}{2}\delta R - f_2$, the seller will not invest with the simultaneous investment regime as part of a subgame perfect equilibrium (SPE) strategy. The seller will, however, invest in the relationship as part of a SPE strategy with the sequential investment regime.*

This proposition mirrors much of the existing literature on the staging of investments with incomplete contracts. For example, Neher (1999) examined financing an entrepreneur overtime in stages rather than funding the entire project up-front. In his model the bargaining power of the financier (vis-a-vis the entrepreneur) is enhanced by the quantity of accumulated physical assets.† Consequently, as the project matures the financier has additional protection from hold-up. The possibility of funding in stages allows projects to proceed that would otherwise not be feasible. In the model presented here, on the other hand, it is assumed that as the project matures contracting becomes possible. If a party can delay their investment until this point in time they can avoid being held up. If the costs of hold-up are sufficiently great as compared with a party's outside opportunities, the sequential regime provides scope for trade that may not have otherwise existed.

Now we consider the case when $\frac{1}{2}\delta R - f_1 \geq 0$ and $\frac{1}{2}\delta R - f_2 \geq 0$. Given this, both parties would enter into the investment relationship if the simultaneous investment

†Physical assets increase the liquidation value of the firm. This enhances the financier's outside option and, as a result, her claim on surplus.

regime were the only option available. It is evident that simultaneous investment always produces greater surplus than sequential investment.† Nevertheless, the seller will act to maximize her own surplus and not to maximize total surplus. As a result, the seller will opt for the sequential regime if:

$$\frac{1}{2}(\delta^2 R - \delta f_2) > \frac{1}{2}\delta R - f_2 \tag{4.7}$$

despite the fact that total surplus is reduced. Herein lies a potential hold-up problem - the seller will opportunistically opt for sequential investments even though surplus is maximized with simultaneous investment. To distinguish the inefficient timing of investment from the standard hold-up problem we call this practice 'follow-up'. This discussion is summarized in Proposition 4.2.

Proposition 4.2 *Assume that $\frac{1}{2}(\delta^2 R - \delta f_2) - f_1 > 0$, $\frac{\delta}{2}R - f_1 \geq 0$ and $\frac{\delta}{2}R - f_2 \geq 0$. If the seller has the choice of whether to invest simultaneously or sequentially and $\frac{1}{2}(\delta^2 R - \delta f_2) > \frac{\delta}{2}R - f_2$, her SPE strategy will be to invest sequentially, reducing total surplus.*

This analysis brings to light another important implication not previously noted in the literature. Although investing over many periods can allow parties to overcome the hold-up problem, it is shown here that the option of staggering investments can be detrimental to overall welfare.†

Now consider the effect of the sequential regime on the buyer's incentive to invest. Sequential investment puts the buyer at a disadvantage as his sequential payoff is necessarily less than his simultaneous payoff. From equations 4.1 and 4.5, $\frac{1}{2}(\delta^2 R - \delta f_2) - f_1 < \frac{1}{2}\delta R - f_1$. The buyer will be willing to enter into the specific

†With discrete investments f_1 and f_2 are unchanged between both regimes. The only effect of a sequential regime is that it further delays the receipt of surplus one additional period from the start of the project. Consequently, if $\delta < 1$ the sequential regime produces a smaller ex ante return.

†In the bargaining literature it has been known for some time that the addition of extra potential bargaining periods can reduce welfare. For example, Fudenberg and Tirole (1983) showed that the addition of extra period in a bargaining game with asymmetric information did not necessarily increase welfare for a bargaining game with only one potential bargaining period.

relationship, despite the inevitable follow-up, if

$$\frac{1}{2}(\delta^2 R - \delta f_2) - f_1 \geq 0. \tag{4.8}$$

On the other hand, if

$$\frac{1}{2}(\delta^2 R - \delta f_2) - f_1 < 0 \tag{4.9}$$

the buyer will not be willing to enter. In this case, the follow-up problem is sufficiently great that the buyer's outside option is more attractive than entering into the relationship.

If the buyer's return from simultaneous investments exceeds his outside option, but the sequential payoff did not, the seller would be better off if they could commit to invest simultaneously. If the seller could guarantee she would invest simultaneously the buyer would opt into the relationship, and both parties would be better off. When the seller cannot commit, the buyer will opt out of the relationship and the seller will suffer as trade between the parties will not occur. Proposition 4.3 summarizes this discussion.

Proposition 4.3 *If* $\frac{1}{2}\delta R - f_1 > 0 > \frac{1}{2}(\delta^2 R - \delta f_2) - f_1$ *and* $\frac{1}{2}(\delta^2 R - \delta f_2) > \frac{1}{2}\delta R - f_2 > 0$*, the buyer will only be willing to invest with the simultaneous regime. The seller will invest sequentially in any SPE in which investment occurs. Anticipating this, the SPE strategy of the buyer will be to not invest. As a result, the surplus of the seller is reduced by having the option of a sequential regime of investment.*

This is a similar result to Grout (1984) who argued that a union would be better off if it could commit not to opportunistically renegotiate after the firm has sunk its investment.

The model presented here also provides a context in which parties can endogenously opt for an incomplete contract. The parties will opt for a (more) incomplete contract here where the loss of total surplus, or the cost of writing a contract, exceeds the benefits from the avoiding hold-up. To see this, again assume that the buyer must invest at the start of the project, but that the seller can opt to invest at the same time as the buyer or sequentially. Further, assume that the buyer will always enter the relationship as $\frac{1}{2}(\delta^2 R - \delta f_2) - f_1 > 0$. The seller will choose to invest simultaneously when $\frac{1}{2}\delta R - f_2 > \frac{1}{2}(\delta^2 R - \delta f_2)$. Despite the option of more complete contracting, the seller chooses to invest with an incomplete contract. These findings are summarized in the following proposition.

Proposition 4.4 *If $\frac{1}{2}(\delta^2 R - \delta f_2) - f_1 > 0$, $\frac{1}{2}(\delta^2 R - \delta f_2) > 0$ and $\frac{1}{2}\delta R - f_2 > 0$, the seller's SPE strategy is to opt for an incomplete contract by investing simultaneously, provided $\frac{1}{2}\delta R - f_2 > \frac{1}{2}(\delta^2 R - \delta f_2)$.*

The comparative statics can be examined when the seller is just indifferent between investing simultaneously or sequentially.† These comparative static results show that the seller is more likely to opt for the incomplete contract when: f_2 is low; and the surplus is high. The seller's incentive to adopt the (more) incomplete regime is decreasing as she becomes more patient, provided the discount factor is sufficiently high.† These results are intuitive. The benefit of the sequential regime to the seller is declining as she becomes more impatient, provided δ is sufficiently large. Further, the net surplus of the seller is the difference between her share of the surplus and the investment costs she has to pay: when f_2 is small there is less benefit sharing this cost with the buyer.

This section examined how the timing of investments can act as a potential source of hold-up. It was shown that if a party can choose to invest prior to or after renegotiation the other party can be held-up by the timing of investment. This reduces the incentive for that party to invest and, in the extreme, prevents surplus enhancing transactions from taking place. The model is sufficiently flexible, however, to also be able to show the potential benefits of sequencing investment.

†Let $W = [\frac{1}{2}\delta R - f_2] - \frac{1}{2}(\delta^2 R - \delta f_2)$. Comparative statics can be calculated for changes in parameter values when $W \approx 0$. Thus, $\frac{\partial W}{\partial f_2} = \delta/2 - 1 < 0$ and $\frac{\partial W}{\partial R} = \frac{1}{2}\delta(1-\delta) > 0$. With respect to δ, $\frac{\partial W}{\partial \delta} = \frac{1}{2}(R + f_2) - \delta R$. If $\delta < \frac{1}{2}(1 + \frac{f_2}{R})$, $\frac{\partial W}{\partial \delta} > 0$. If $\delta > \frac{1}{2}(1 + \frac{f_2}{R})$, $\frac{\partial W}{\partial \delta} < 0$.

†There is an additional effect of δ due to the discounting structure in the staged investment regime. That is, $\delta < \frac{1}{2}(1 + \frac{f_2}{R})$ the incentive to opt for the simultaneous regime is increasing as δ increases, whereas if $\delta > \frac{1}{2}(1 + \frac{f_2}{R})$, the incentive to opt for an incomplete contract - with the simultaneous regime - is decreasing in δ.

Sequencing allows contracts to become complete: this protects the party investing second from being held-up, and consequently encourages investment by that party.

4.4 Leading or following investment game

Up until this point it has been assumed that the seller has the option to adopt the sequential regime. What happens if either of the individuals can be the party that invests first? If either party can invest first, it follows that either agent could wait until the other player has invested so that they can invest when contracts are complete. When there is an advantage of investing after the other party has sunk their investment (a follower advantage), the two players may vie to invest second.

To investigate this, assume the parties are identical, so that $f_1 = f_2 = f$. Further, assume that an investment by either individual would allow contracting to be feasible. If $\frac{1}{2}(\delta^2 R - \delta f) > \frac{\delta}{2}R - f$, both individuals would prefer to invest second.† Let us consider this case in more detail.

First, consider when there are just two potential investment periods in which the project can be completed and that $[\frac{1}{2}(\delta^2 R - \delta f) - f] < 0$. In this case neither party will be willing to invest first. Moreover, a contract on the timing of investments coupled with some up-front compensation is unlikely to resolve the hold-up problem. Given the non-verifiability of investment, a contract written on the timing of investment is unenforceable. For example, after the compensation payment has been made the recipient can simply trigger renegotiation without fear of sanction. As a result, trade is unlikely to proceed in this case. Adding additional periods will not change this outcome.

Second, consider when the payoff with sequential investment for the lead investor - who invests at $t = 1$ - is positive: that is, $[\frac{1}{2}(\delta^2 R - \delta f) - f] > 0$. To explore the strategies the players will adopt initially, consider when there are exactly two periods remaining in which the project can be completed. The choice for each player is then

†When $\frac{1}{2}(\delta^2 R - \delta f) < \frac{\delta}{2}R - f$ the return from simultaneous investment exceeds the maximum possible sequential payoff and both parties will invest at $t = 1$.

to invest immediately at $t = 1$ or to wait and invest in the final period at time $t = 2$. As surplus from simultaneous investments is greater than the no-trade option, if the game reaches $t = 2$ both agents would invest if they had not previously done so. The normal form of this game is illustrated in Figure 4.3. In the figure, I represents investing at $t = 1$ and W waiting and investing at $t = 2$. The payoff for the buyer is written in the left of each box in the matrix and the seller's on the right.

		Seller	
		I	W
Buyer	I	$\frac{\delta}{2}R - f$, $\frac{\delta}{2}R - f$	$\frac{1}{2}(\delta^2 R - \delta f) - f$, $\frac{1}{2}(\delta^2 R - \delta f)$
	W	$\frac{1}{2}(\delta^2 R - \delta f)$, $\frac{1}{2}(\delta^2 R - \delta f) - f$	$\delta(\frac{\delta}{2}R - f)$, $\delta(\frac{\delta}{2}R - f)$

Figure 4.3: Normal form for two period game

As

$$\frac{1}{2}(\delta^2 R - \delta f) > \frac{\delta}{2}R - f \tag{4.10}$$

and

$$\delta(\frac{\delta}{2}R - f) > \frac{1}{2}(\delta^2 R - \delta f) - f \tag{4.11}$$

both players have a dominant strategy of delaying and investing at time $t = 2$. This is a version of prisoners' dilemma: surplus is maximized if both players invest simultaneously at $t = 1$, so as to avoid the additional costs of delay, but the only Nash equilibrium in this game is that each player will delay investing.

This artifact of the equilibrium arises as a result of the short time horizon. Now consider the case when there are three potential investment periods.† The choice of each player initially is to invest immediately at $t = 1$ or to wait. If both players opt to invest at $t = 1$ the project is completed in the first period and the payoffs are unchanged from the two-period horizon game when the project is completed immediately. Similarly, if at $t = 1$ the buyer invests but the seller does not, she will invest at $t = 2$.† In this case the payoffs are unchanged from the two-period example above. Similarly, if the seller invests at $t = 1$ and the buyer invests at time $t = 2$, the payoffs are also unchanged from two-period game. The only payoff that is altered is when both players opt to not invest at $t = 1$. In this case, the parties again face a two period potential investment horizon (at times $t = 2$ and $t = 3$). From above, the equilibrium in this two-period horizon game is that both players wait until the last period to invest. Consequently, the payoff in the three-period horizon game when both parties do not invest at $t = 1$, is the two-period payoff discounted for the extra period - that is, $\delta^2(\frac{\delta}{2}R - f)$. Provided

$$\delta^2(\frac{\delta}{2}R - f) > \frac{\delta}{2}(\delta R - f) - f \tag{4.12}$$

the dominant strategy remains to not invest at $t = 1$ for both players.

As more potential trading periods are added a similar adjustment of the payoffs continues. Figure 4.4 shows the normal form of the game with n potential bargaining periods.

At some point, say when the potential horizon has n periods, the payoff from not investing when the other player also does not invest becomes less than the payoff from choosing to invest immediately. This occurs when

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†Note, as above a maximum of two periods is needed to complete the project.

†As contracting is possible at time $t = 2$, there is no advantage to the seller to wait until time $t = 3$ as this will merely delay her receiving her payoff an extra period, without increasing her claim on surplus.

		Seller	
		I	W
Buyer	I	$\frac{\delta}{2}R - f$, $\frac{\delta}{2}R - f$	$\frac{1}{2}(\delta^2 R - \delta f) - f$, $\frac{1}{2}(\delta^2 R - \delta f)$
	W	$\frac{1}{2}(\delta^2 R - \delta f)$, $\frac{1}{2}(\delta^2 R - \delta f) - f$	$\delta^{n-1}(\frac{\delta}{2}R - f)$, $\delta^{n-1}(\frac{\delta}{2}R - f)$

Figure 4.4: Normal form for n period game

$$\delta^{n-1}(\frac{\delta}{2}R - f) < \frac{1}{2}(\delta^2 R - \delta f) - f < \delta^{n-2}(\frac{\delta}{2}R - f). \tag{4.13}$$

When the potential bargaining horizon is n periods there is no longer a dominant strategy for each player: each player will play a mixed strategy between investing immediately and waiting. The intuition is that, when there is a long potential time horizon, the players know that stalling until the end of the potential horizon is of little benefit, as there is a sufficiently large number of periods that the payoff from waiting that long is relatively small. This provides an incentive to invest immediately. However, there is also a potential dividend from waiting on the chance that the other party invests immediately. The players are in a coordination game in that period: each party wants the project to go ahead immediately but both investors would prefer to follow rather than lead.[†] Also note that the game with $n+1$ potential investing periods may return to a prisoners' dilemma game. This arises because the coordination game with n periods is the outcome of waiting in the first of the $n+1$ periods. The payoff of this coordination game might be higher than $\frac{1}{2}(\delta^2 R - \delta f) - f$, which again creates a dominant strategy to wait. The game could switch between a prisoners' dilemma and a coordination game as potential investment periods are added. As an illustration of this, consider the following example.

Example 4.1 *The following example shows the possibility of switching between a prisoners' dilemma and a coordination game when there are many potential investment periods.*

Let $\delta = 0.9$, $f = 10$ and $R = 100$. Figure 4.5 illustrates the normal form game of the investment decision for both parties when there are $n = 2, 3, \ldots$ potential investment periods.

From Figures 4.3 and 4.4 the payoffs are $A = 35$, $B = 26$, $C = 36$ and $D = \delta^{n-1}35$. If $n = 2$ and 3, the SPE strategy of both players is to wait - this is a prisoners' dilemma. When $n = 4$, in the first potential investment period both players adopt a mixed strategy. This is a coordination game.

Now we show that for $n = 5$ the game reverts to a prisoners' dilemma. If the buyer chooses actions I and W with probabilities α and $1-\alpha$ respectively, while the seller mixes between I and W with probabilities β and $1-\beta$, the expected return of the buyer is

$$A\alpha\beta + B\alpha(1-\beta) + C\beta(1-\alpha) + D(1-\alpha)(1-\beta). \tag{4.14}$$

To get a Nash Equilibrium in mixed strategies we find β such that the payoff to the

[†]Note, this is not a typical coordination game. Instead, it is similar to what Binmore (1992) described as an Australian Battle of the Sexes; the two parties want to coordinate to be where the other player is not. Further, this game is not Matching Pennies as it is not a zero sum game.

		Seller	
		I	W
Buyer	I	A, A	B, C
	W	C, B	D, D

Figure 4.5: Normal form for n period game

buyer does not depend on α, in other words

$$\beta = \frac{B - D}{B + C - A - D}. \tag{4.15}$$

Similarly, for the payoff of the seller not to depend on β it must be the case that

$$\alpha = \frac{B - D}{B + C - A - D}. \tag{4.16}$$

The payoff to the buyer from playing this mixed strategy is

$$D + (C - D)\frac{B - D}{B + C - A - D} = \frac{BC - AD}{B + C - A - D} = B + \frac{(A - B)(B - D)}{B + C - A - D}. \tag{4.17}$$

Because $C > A > B > D$ this payoff is always greater than B and, provided the discount factor is sufficiently high, the game returns to a prisoners' dilemma in period $n = 5$. As $\delta = 0.9$ in this specific example, the relevant payoff for period $n = 5$ - $\frac{BC-AD}{B+C-A-D}\delta$ - is greater than B. On the other hand, if δ were small enough we could end up in the coordination game $\forall\ n \geq 4$. Thus, it is not possible to discern the exact structure of the game when $n \rightarrow \infty$ without knowledge of the precise parameter values. □

Two points are important here. First, if both parties have the opportunity to wait until after the other has invested, strategic behavior can reduce total surplus. Second, if for technical reasons, as assumed above, one player (the buyer) must invest at the start of the project, the potentially damaging coordination game regarding which party is to invest first is avoided. This suggests technical differences in the individuals, that determine which of the parties must invest at the beginning of the project, may help overcome some of the problems generated by the timing of investments and follow-up.

The issue of which party must invest first could be resolved naturally when the parties have differing investment costs.† Again assume that the investment costs and outside options are f_i for $i = 1, 2$, as in section 4.3. However, now assume that there is no specified order of investments (that is, either party can invest first to start the project). If $[\frac{1}{2}(\delta^2 R - \delta f_2) - f_1] > 0$, the buyer will be willing to enter the relationship regardless of which regime eventuates. Further, assume that if the seller's cost of investment (f_2) is sufficiently high as to ensure that $\frac{\delta}{2}(\delta R - f_2) > 0 > \frac{1}{2}\delta R - f_2$. In this case the seller would never invest first. She would be willing, however, to contract with the buyer after he has made his investment. Once again the option of sequential investments improves welfare - it allows for a trade opportunity that would not otherwise occur. The differing opportunity costs of the parties make it clear which party is to invest at $t = 1$: the party with the smallest investment cost should invest first. This prediction accords with what is observed with venture capital projects.† It is often the case that the financier waits until the entrepreneur, the party with the smaller opportunity cost, has already made their investment and

†Of course, investment costs could include opportunity costs.

†See Gompers and Lerner (1999) for a discussion of venture capital.

the project is underway before committing to the project. Sequencing of investment in this case affords the financier the protection from hold-up needed to encourage participation.†

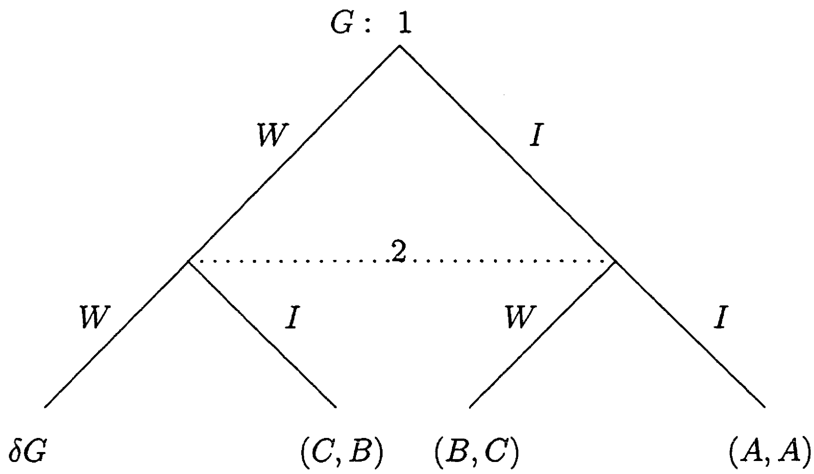

Figure 4.6: Extensive form for infinite horizon game

Finally, consider when the parties face a potentially infinite-horizon.† The extensive form game is illustrated in Figure 4.6. Here, player 1 (for example the buyer) can choose to invest immediately (I) or can choose to wait (W). At the same time player 2 (the seller) has the strategic options of investing (I) and not investing and waiting (W). There are three stationary SPE in this game. The first involves player 1 playing the following strategy: do not invest in the first period; do not invest in the second period unless player 2 invested in period $t = 1$; do not invest in the third period unless player 2 invested in period $t = 2$; and so on. Player 2 plays the following strategy: invest in the first period; if not in the first, invest in the second; if not in the second, invest in the third; and so on. In this equilibrium player 2 invests immediately and player 1 follows up with their investment in the

†Differing discount factors between the two parties may also help to resolve which party should invest first. In this case the more patient player may be willing to invest first. The second player with the lower discount factor, is consequently afforded the benefits of investing a period closer to the receipt of surplus.

†Again assume the parties are identical.

next period. Neither player has an incentive to deviate in any subgame. Player 1 receives the highest payoff possible in this game - C - while player 2 receives a payoff of B. If player 2 deviates to invest in the second period, she will receive a payoff of δB, ruling out any possibility of a profitable deviation. A symmetrically equivalent equilibrium exists in which player 1 invests immediately and player 2 invests in the second period.

A stationary mixed strategy equilibrium also exists. In this equilibrium both parties invest with some positive probability. For example, player 1 invests immediately with probability α and player 2 invests immediately with probability β. If at least one player invests the entire investment process will last no longer than two periods and the game will end. The payoffs to each player are outlined in Figure 4.6. However, if both parties do not invest in the first period, which occurs with probability $(1-\alpha)(1-\beta)$, the players return to an identical situation, only one period in the future. In this continuation game the players will again adopt the same strategies. As a result the expected payoff of each player are exactly the same as at $t=1$, however, they are discounted from the delay of one period. This symmetric mixed strategy equilibrium is always feasible, for any parameters where $C > A > B$, as summarized in Proposition 4.5.

Proposition 4.5 *A stationary mixed strategy SPE always exists in the infinite horizon investment game, provided $C > A > B$.*

Proof. Example 4.1 calculated the payoff of an agent from playing a mixed strategy: this payoff is given by equation 4.17. Given the stationarity of strategies, any SPE requires this payoff multiplied by δ to equal D, yielding the following equation:

$$D^2 + D(A(1-\delta) - B - C)) + \delta BC = 0. \qquad (4.18)$$

This quadratic equation has two solutions: the first is less than B, the second is greater than B. The first is feasible as a solution to this problem, while the second is not, as either player will only adopt a mixed strategy when $D < B$. (If $D > B$, both parties have dominating strategy to wait.) As one solution is always less than B, a mixed strategy always exits. □

Note that all the equilibria produce lower ex ante welfare than the first-best outcome.

4.5 Concluding comments

This chapter develops a model in which two parties can invest in a mutually beneficial project together at the same time (simultaneous investment) or they can choose to have the investments made one after the other (sequential investment). It is assumed that contracting on any future investment becomes possible after some investment has been made, as it allows the project to become more clearly defined. Consequently, the advantage of the sequencing of investments is it allows the party that has delayed making their investment to avoid being held-up. The disadvantage of staging is that it reduces the payoff of the first-mover. Sequencing of investment also lengthens the time from the start of the project until the returns are realized, reducing the ex ante value of total surplus when parties discount future returns.

Much of the emphasis in the existing literature has focused on how staging investments can improve welfare when there are incomplete contracts or when parties are unable to commit. In the model presented in this paper it is demonstrated that, in some cases, the option of sequencing investments can reduce welfare. It is shown that under certain conditions a party will opportunistically opt for the sequential regime, reducing total surplus. We interpret this possibility as a new form of hold-up and term it 'follow-up'. Moreover, in some cases the mere possibility that investment can be made sequentially may discourage investment by one party, preventing trade from occurring and reducing welfare of both players.

Interesting dynamics can arise if both players prefer to follow rather than make their investment before (or concurrently with) the other party. With just two potential investment periods, both players have a dominant strategy to not invest in the first period and wait to invest in the second period. This is a version of the prisoners' dilemma. As the potential investment horizon is extended, the payoff from waiting is discounted so that, with an investment horizon of a certain length, the players adopt a mixed strategy of investing immediately or waiting. This is a coordination game. As the investment horizon is extended even further from this point, the players may again return to a prisoners' dilemma, in which they have a dominant strategy to not invest until they reach the investment period in which they are in the coordination game.

CHAPTER 5

Timing of investments, hold-up and total welfare

5.1 Introduction

Recently several papers have shown how sequencing investment can help alleviate the hold-up problem. For example, Neher (1999) demonstrated that staged financing could improve the bargaining position of a financier in the event of default, allowing a project to proceed when an entrepreneur was unable to commit not to renegotiate. De Fraja (1999) considered how the Stackelberg-type sequencing of investment could overcome hold-up when the first party makes a general investment, then proposes a contract that makes him the residual claimant of surplus.[†] In addition, Admati and Perry (1991) showed two parties can overcome the free-rider problem by financing a public good in stages.

There are costs involved with sequencing investments, however. For example, sequencing a project lengthens the time required for it to be completed, increasing the costs of delay if parties discount the future. In addition, sequencing may improve the relative bargaining position of one party relative to the other: while this may increase the stronger agent's incentive to invest, it can decrease the investment of the weaker agent. This may occur, for example, if the second investor's bargaining power is improved after the first player has sunk their investment.

To explore these issues this chapter develops a theoretical model that compares

[†]The first investment is general because, although it can be industry-specific, it is not relationship specific in the traditional sense (see Malcomson 1997).

the relative efficiency of simultaneous and sequential investment under the presence of hold-up. In the model two parties are both required to make a specific investment to complete a project. Two alternative timing regimes are possible. First, both parties can invest simultaneously. In the usual manner, as contracts are initially incomplete renegotiation occurs after both parties have sunk their investments. Following renegotiation, the project is completed and the payoffs are realized. Alternatively, following De Fraja (1999), the parties can invest one after the other in a Stackelberg-type arrangement. In this chapter it is assumed that the resolution of incompleteness is facilitated by completion of some aspect of the project. More specifically, after the first investment is sunk contracting on the second investment becomes possible. This set-up is similar to Grossman and Hart (1986) who assumed that contracting became possible after the two parties made their initial investment. Similarly, in Neher (1999) investment facilitated more complete contracting by converting intangible human capital into physical assets. As a consequence, with the sequential regime, after the first player has invested, the parties will renegotiate and the second party will invest according to the agreed contract, completing the project.

Several important results arise from this simultaneous versus sequential investment model. First, the chapter investigates the relative efficiency of the two alternative investment regimes. When the investments are independent the model identifies three basic trade-offs between the regimes:

- The sequential system enlarges (relative to simultaneous investments) delay costs by increasing the length of time before the project matures.
- The sequential system reduces the first player's incentive to invest, vis-a-vis the simultaneous system, because of the longer time between when his investment is made and when the returns are realized.
- The sequential system enhances the incentive for the second player to invest efficiently as they do not suffer hold-up, which is not the case with simultaneous investments.

The ultimate impact on total surplus is a combination of these trade-offs. We show that under different circumstances either regime of investment maximizes welfare. Moreover, despite the simplicity of the model, no simple relationship between the welfare effects of the two regimes exists, as there is no restriction on how these three trade-offs interact. However, given that the simultaneous regime encourages the first player to invest, if this player's contribution is relatively more important than the other player's contribution the simultaneous regime is preferred. In the same way, the sequential regime is preferred when the second investor's contribution is relatively more important, provided both players are sufficiently patient. Similarly, if a party is not responsive to the incentives provided by one timing regime, the regime that maximizes the other party's incentive to invest is preferred. For example, when the first investor is not responsive to the additional incentive provided by the simultaneous regime, the sequential regime generates a higher level of surplus.† On the other hand, if the second player is not responsive to the additional incentives to invest provided by the sequential regime, the simultaneous regime is preferred. These predictions are similar in nature to the property-rights predictions of Hart (1995), although the model is somewhat more general in that a player may voluntarily forgo the advantages of sequencing of investment (their property right) in order to encourage the other party to invest more. In this way the parties can opt for a (more) incomplete contract by choosing to invest simultaneously. The model is also extended in several other ways, for example by considering these trade-offs when the two investments are strategic complements and substitutes.

Second, we show that the possibility of investing sequentially does not always improve welfare. As it turns out, flexibility in the timing of investment can act as an additional form of hold-up. We call this kind of hold-up 'follow-up'. This occurs when both parties should invest simultaneously at the start of the project in order to maximize surplus, but there is an incentive for the second party to wait until after the first player has sunk their effort before they follow-up with their

†Again this assumes both parties are sufficiently patient.

own investment.[†] In this case, the first party will anticipate that the second party will opt for the sequential regime. The first party will then reduce their investment accordingly, decreasing total surplus. In the extreme this additional form of hold-up will prevent a potential surplus-enhancing project from proceeding. A similar result arises in the model presented in the previous chapter.

5.2 The model

There is a potentially profitable relationship between two parties that, for convenience, we label as a buyer and a seller. Specifically, if the buyer and seller invest I_1 and I_2 respectively the two parties share surplus R. We assume that total surplus is a function of both investments where $R(I_1, I_2)$ is two times differentiable, non-decreasing in both variables and concave; that is $R_i' = \partial R(I_1, I_2)/\partial I_i \geq 0$, $R_{ii}'' = \partial^2 R(I_1, I_2)/\partial I_i^2 \leq 0$ for $i = 1, 2$ and $R_{11}'' R_{22}'' - (R_{12})^2 \geq 0$, as summarized by Assumption 5.1.

Assumption 5.1 $R_i' = \partial R(I_1, I_2)/\partial I_i \geq 0$, $R_{ii}'' = \partial^2 R(I_1, I_2)/\partial I_i^2 \leq 0$ *for* $i = 1, 2$ *and* $R_{11}'' R_{22}'' - (R_{12})^2 \geq 0$ *where* $R_{12} = \partial^2 R(I_1, I_2)/\partial I_1 \partial I_2$.

Two alternative timing arrangements are considered. First, both players invest simultaneously at time $t = 1$, as shown in Figure 5.1. Initially, contracting on either investment is not possible; consequently renegotiation will occur after both investments are sunk.[†]

Figure 5.2 outlines the timing of the alternative investment regime. In this regime the buyer invests I_1 at time $t = 1$ prior to when contracting is possible. However, this investment makes the contracting process possible, so having observed I_1 the two parties renegotiate and contract on I_2. It is only after this that the seller makes her investment I_2. This occurs at time $t = 2$. After both investments have been made, surplus is realized and the payoffs to each party are made. Both parties

[†]'Follow-up' can occur in addition to the regular hold-up of investment.

[†]The renegotiation process is discussed below.

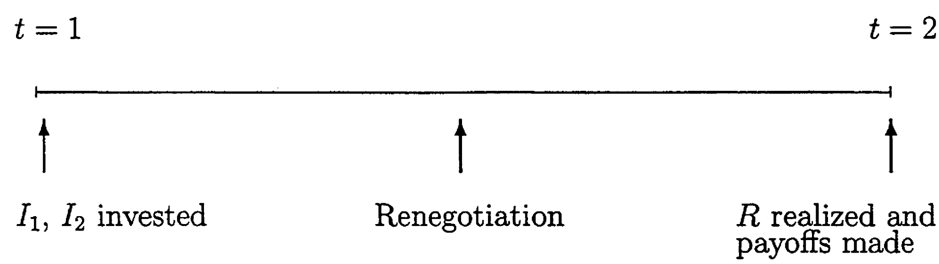

Figure 5.1: Simultaneous investment

discount future payoffs and costs with a constant discount factor per period of $\delta \in (0, 1]$. As a result, at the beginning of the project the returns from the simultaneous regime are valued at δR, while the returns with the sequential investment are worth $\delta^2 R$.

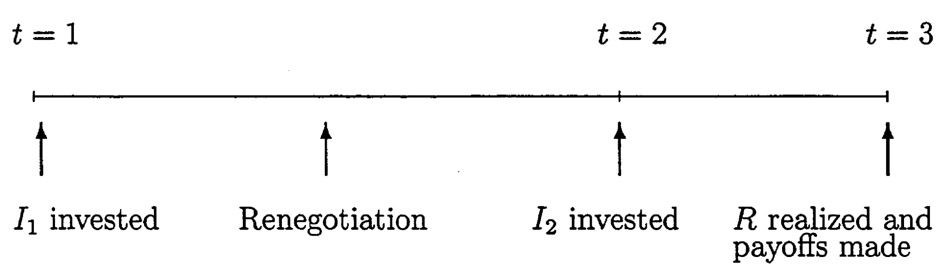

Figure 5.2: Sequential investment

The investments of both parties are sunk and completely specific to the relationship, in that they are worth zero outside the relationship. R is only available at the completion of the project and investment in the relationship is always efficient. Further, it is assumed that each party's outside option is normalized to zero.

Although investments are unverifiable ex ante, once the buyer's investment has been sunk the project becomes tangible, allowing subsequent investment to be verifiable. On the contrary, the surplus generated by the project is always unverifiable. This prevents the parties writing surplus sharing agreements. Further, as in Hart and Moore (1988) and MacLeod and Malcomson (1993), the two parties cannot vertically integrate to overcome their hold-up problem.

When the parties renegotiate they must decide how to split the available surplus. We adopt a reduced-form bargaining solution in which each party receives one-half of the available surplus.† Unlike many incomplete-contracts models, the results in this chapter are not sensitive to the bargaining solution used.

5.3 Simultaneous and sequential investments

As contracts are incomplete when investments are made simultaneously both parties know that renegotiation will occur. Consequently, they adjust their investments from the first-best level accordingly. The buyer chooses I_1 in order to maximize

$$\max_{I_1} \frac{\delta}{2} R(I_1, I_2) - I_1. \tag{5.1}$$

Here, the returns are discounted by δ, because they are only available after one period. Renegotiation occurs after both investments have been sunk; as a consequence each party anticipates receiving one half of the available surplus. The first-order condition for the buyer is

$$R_1' = \frac{2}{\delta}. \tag{5.2}$$

The seller faces a similar decision choosing her level of I_2. She will set I_2 to maximize

$$\max_{I_2} \frac{\delta}{2} R(I_1, I_2) - I_2, \tag{5.3}$$

†This reduced form bargaining solution can be thought of as relating to an extensive form bargaining game.

which yields the first-order condition of

$$R_2' = \frac{2}{\delta}. \tag{5.4}$$

Let the buyer's and seller's investment choices with the simultaneous regime be denoted by $\widehat{I}_1$ and $\widehat{I}_2$ respectively. These values solve system of equations 5.2 and 5.4. The solutions are unique because of Assumption 5.1.

Now consider when the investments are made sequentially. With the sequential regime the buyer invests I_1 at time $t = 1$. Following renegotiation, at time $t = 2$ the seller chooses I_2. Consequently, the buyer sets I_1 to maximize

$$\max_{I_1} \frac{\delta}{2}[\delta R(I_1, I_2) - I_2] - I_1. \tag{5.5}$$

The first-order condition for this problem is

$$R_1' = \frac{2}{\delta^2}. \tag{5.6}$$

The seller, who sets her investment level after observing I_1 and renegotiating with the buyer will maximize

$$\max_{I_2} \frac{\delta}{2}[\delta R(I_1, I_2) - I_2]. \tag{5.7}$$

The first-order condition for this maximization problem is

$$R_2' = \frac{1}{\delta}. \tag{5.8}$$

Let the buyer's and the seller's levels of investment be $\widetilde{I}_1$ and $\widetilde{I}_2$ when contracts are made sequentially. These values are the solution to the system of equations 5.6 and 5.8. Again, the solutions are unique because of Assumption 5.1.

Finally, define I_1^* and I_2^* as the first-best level of investment with the simultaneous regime. Similarly, let I_1^{**} and I_2^{**} be the first-best investment levels with the sequential regime.[†]

5.4 Timing of investment and total welfare

As it turns out, very little can be said about the trade-off between simultaneous and sequential investments when functions are general. Following Hart and Moore (1988), to explore this issue further assume that each investment has no influence on the marginal productivity of other player's investment; that is, $R_{12} = 0$, as stated in the following assumption.[†]

Assumption 5.2 $R''_{12} = 0$.

Remark 5.1 *If* $R''_{12} = 0$, *it follows that* $R = f_1(I_1) + f_2(I_2)$, *where* $f'_i > 0$ *and* $f''_i \leq 0$ *for* $i = 1, 2$.

In this framework three separate effects can be isolated that, when combined, give the relative advantage of either timing regime. First, consider the costs of delay. Let the total surplus ex ante with simultaneous investment be S_{sim} and the total surplus ex ante when investment is sequential be S_{seq}. For two fixed levels of $\overline{I}_1$ and $\overline{I}_2$

$$S_{\text{sim}} = \delta R(\overline{I}_1, \overline{I}_2) - \overline{I}_1 - \overline{I}_2 > \delta^2 R(\overline{I}_1, \overline{I}_2) - \overline{I}_1 - \delta \overline{I}_2 = S_{\text{seq}}. \tag{5.9}$$

As sequential investment delays the payoff an extra period, the surplus from simultaneous investment is greater than with sequential investments when I_1 and I_2 are fixed: the costs of delay always favor simultaneous investment. Further, the relative payoff of simultaneous investments is increasing as the players become more impatient. This effect is summarized below.

[†]The first-best level of investments solve the following system of equations: $R_1^* = R_2^* = \frac{1}{\delta}$ for the simultaneous regime; and $R_1^{**} = \frac{1}{\delta^2}$ and $R_2^{**} = \frac{1}{\delta}$ for the sequential one.

[†]$R''_{12} = 0$ could arise when an investment by the buyer increases his benefit from trade whereas investment by the seller reduces her costs. Although they do not affect one another, each investment increases the potential surplus available to be split upon renegotiation. A similar assumption is made by Hart and Moore (1988).

Effect 5.1 *The costs of delay reduce the surplus generated by sequential investment relative to the surplus with simultaneous investments.*

Second, consider the buyer's investment decisions under both regimes. Examining the first-order conditions in equations 5.2 and 5.6, $\widehat{R}_1' = \frac{2}{\delta} \leq \widetilde{R}_1' = \frac{2}{\delta^2}$. Given the assumption of concavity and monotonicity of R:

$$\widehat{I}_1 > \widetilde{I}_1. \tag{5.10}$$

The sequential investment regime delays the collection of returns to the buyer: this reduces the incentive for the buyer to invest.†

Effect 5.2 *Relative to sequential investment, the simultaneous regime increases the incentive for the buyer to invest in I_1.*

For the seller the relative incentives to invest with simultaneous and sequential investments are given by equations 5.4 and 5.8. Again, because of Assumption 5.1,

$$\widehat{I}_2 < \widetilde{I}_2. \tag{5.11}$$

With simultaneous investment the seller is held-up. With sequential investment, however, the seller invests after renegotiation, thus avoiding any hold-up problems. In fact, the sequential investment level chosen by the seller equals the first-best level; so that $\widetilde{I}_2 = I_2^{**}$. This is the advantage of the sequential regime over simultaneous investment. Effect 5.3 summarizes this discussion.

Effect 5.3 *The sequential investment regime increases I_2 to its first-best level.*

Effect 5.2 states that the simultaneous regime increases I_1. Effect 5.3 suggests that the sequential regime increases I_2. To assess the impact of an increase in either investment on total welfare, isolated from the costs of delay, consider S_{sim} relative

†Note that both $\widehat{I}_1$ and $\widetilde{I}_1$ are below the first-best level. With simultaneous investments $\widehat{R}_1' = 2/\delta > R_1' = 1/\delta$, meaning that $\widehat{I}_1 < I_1^*$. Similarly, with sequential investment, $\widetilde{R}_1' = 2/\delta^2 > R_1' = 1/\delta^2$, meaning that $\widetilde{I}_1 < I_1^{**}$.

to an augmented S_{seq}, termed U_{seq}, that has the same discount structure as the simultaneous system. U_{seq} ignores the additional discounting of R and of I_2 that occurs because of the additional period. In this case:

$$S_{\text{sim}} = \delta f_1(\widehat{I}_1) - \widehat{I}_1 + \delta f_2(\widehat{I}_2) - \widehat{I}_2. \tag{5.12}$$

where the level of investments are determined by equations 5.2 and 5.4. Similarly, using equations 5.6 and 5.8

$$U_{\text{seq}} = \delta f_1(\widetilde{I}_1) - \widetilde{I}_1 + \delta f_2(\widetilde{I}_2) - \widetilde{I}_2. \tag{5.13}$$

The relative incentives to invest for the seller and buyer are summarized in the following lemma.

Lemma 5.1 $\delta f_1(\widetilde{I}_1) - \widetilde{I}_1 < \delta f_1(\widehat{I}_1) - \widehat{I}_1$, *and* $\delta f_2(\widetilde{I}_2) - \widetilde{I}_2 > \delta f_2(\widehat{I}_2) - \widehat{I}_2$.

Proof. The first-best investment level of I_1, derived from $\delta f_1(I_1) - I_1$, occurs when $f_1' = \frac{1}{\delta}$. This level of investment is termed I_1^*. For $I_1 < I_1^*$, $f_1'(I_1) \geq \frac{1}{\delta}$ because $f_1''(I_1) \leq 0$. For $I_1 < I_1^*$, $[\delta f_1(I_1) - I_1]' \geq 0$, hence $\delta f_1(I_1) - I_1$ is a non-decreasing function $\forall\ I_1 \in [0, I_1^*)$, which means $\delta f_1(\widetilde{I}_1) - \widetilde{I}_1 < \delta f_1(\widehat{I}_1) - \widehat{I}_1$. A similar argument applies to I_2. $\square$

Lemma 5.1 indicates that increasing I_1 towards its first-best level always increases the surplus it generates. The same argument applies to I_2. From Lemma 5.1 we can say that the surplus generated by I_1 is greater with the simultaneous regime. Similarly, the surplus generated by I_2 is greater with the sequential regime.

In terms of total surplus, the ultimate trade off between simultaneous and sequential systems depends on these three effect: costs of delay incurred with the sequential regime favor simultaneous investments; given the additional hold-up, the sequential system reduces the buyer's contribution to total surplus as compared with the simultaneous system; and, finally, the sequential system increases the incentive for the seller to invest, increasing her contribution to total surplus. Two of these effects work in favor of the simultaneous system and one works in favor of the sequential system. Proposition 5.1 summarizes this discussion.

Proposition 5.1 *There are three factors that affect the total surplus generated by the simultaneous system relative to the total surplus that will be generated by the sequential system: (Effect 5.1) costs of delay favor the simultaneous system; (Effect 5.2) the simultaneous regime increases the buyer's incentive to invest, increasing his contribution to total surplus; and (Effect 5.3) the sequential regime increases the seller's incentive to invest, relative to the simultaneous regime.*

The combined effect of these three effects can be complicated. Note, however, that the three effects each depend on δ: the costs of delaying the return of surplus another period directly relate to δ; the level of I_1 depends on δ as the two relevant first-order conditions are $\widetilde{R}_1' = 2/\delta^2$ and $\widehat{R}_1' = 2/\delta$; and the two first-order condition for the choice of I_2 are $\widetilde{R}_2' = 1/\delta$ and $\widehat{R}_2' = 2/\delta$. However, if $\delta = 1$ the first two of these effects disappear. The only remaining effect is that sequential investment allows the seller to avoid being held-up, increasing her incentive to invest. Thus, if $\delta = 1$, $S_{\text{sim}} < S_{\text{seq}}$. As R is a continuous function it follows that there is a neighborhood for δ close to 1 where the surplus from sequential investment exceeds the surplus generated with simultaneous investments. This is summarized in the following remark and an example is provided in the Appendix.

Remark 5.2 *There is a small enough ε such that for any $\delta \in (1-\varepsilon, 1]$, $S_{sim} < S_{seq}$; that is, the surplus from sequential investments exceeds that produced with simultaneous investments.*

It is not possible, however, to establish that the relative difference between the surplus from sequential and simultaneous investments is monotonically increasing in δ. With general functions, the relationship between δ, I_1 and I_2 and total surplus, R can be complicated. See the Appendix for a particular example.

Remark 5.3 *No monotonic relationship between the surplus from the simultaneous and sequential systems as δ changes.*

5.5 Hold-up and the choice of investment regime

Thus far we have considered the relative merits of the various timing arrangements in terms of total welfare. The focus shifts here to explore the incentive for the seller, acting in self-interest, to choose the investment regime that does not maximize total surplus. Implicit in this discussion is the assumption that the buyer must invest at the beginning of the project. As a result, only the seller has the opportunity to delay her investment and follow-up the buyer.†

There is a trade-off for the seller when she chooses between the two regimes. As simultaneous system encourages the buyer to invest, this may allow the seller to capture more surplus during renegotiation. However, sequential investment allows the seller to invest without the fear of hold-up. The seller will choose the regime that maximizes her welfare. Where her interests differ sufficiently from the first-best incentives the seller will adopt the 'wrong' system, reducing total welfare.

The seller may find it in her interests to adopt the sequential system when simultaneous investment maximizes welfare. She will not, however, adopt a simultaneous system when the sequential regime maximizes welfare. With inefficient simultaneous investment the seller will lose out on two fronts: first, she will incur hold-up with simultaneous investments; and second, she will be sharing a lower total surplus. Consequently, she will never have any incentive to opt for the simultaneous regime inefficiently.

To further investigate the incentives of the seller assume that $R_{12} = 0$, as in Assumption 5.2, and that the buyer's level investment is invariant to the seller's choice of regime.† Consequently, I_1 can be suppressed, allowing all attention to revolve around the choice about the timing of I_2. The seller will then choose the system (and the level of investment) that maximizes her surplus, regardless of the effect on total welfare.

†See the previous section for a discussion relating to the case when each party can invest either first or last.

†The buyer's investment may be invariant, for example, because he has extreme beliefs about the seller's investment strategy: the buyer could be either naive or pessimistic as to whether the seller will opt for the simultaneous or sequential regime.

With simultaneous investments, total welfare can be written as

$$\delta\widehat{R} - \widehat{I}_2 \tag{5.14}$$

for the seller's choice of investment $I_2 = \widehat{I}_2$, suppressing I_1. The seller will set I_2 to maximize

$$\frac{\delta}{2}\widehat{R} - \widehat{I}_2. \tag{5.15}$$

Denote the seller's objective function under the simultaneous regime as v_1; that is, $v_1 = \frac{\delta}{2}\widehat{R} - \widehat{I}_2$. This allows the total welfare generated with simultaneous investments to be written as $2v_1 + \widehat{I}_2$.

With sequential investments total welfare is

$$\delta^2\widetilde{R} - \delta\widetilde{I}_2, \tag{5.16}$$

while the seller's objective function is

$$\frac{\delta^2}{2}\widetilde{R} - \frac{\delta}{2}\widetilde{I}_2. \tag{5.17}$$

Denote the seller's objective functions under sequential investment as v_2: that is, $v_2 = \frac{\delta^2}{2}\widetilde{R} - \frac{\delta}{2}\widetilde{I}_2$. This means that total surplus generated with sequential investments is $2v_2$.

Now assume these potential payoffs for the seller are also equal so that $v_1 = v_2$. Given $\widehat{I}_2 > 0$, simultaneous surplus will be greater than the surplus from sequential investments. It is possible, however, to perturb v_2 such that $v_2 > v_1$ while it remains true that simultaneous surplus exceeds the surplus with sequential investments, as $2v_1 + I_2 > 2v_2$. In this case the seller will opt for the sequential regime even though total surplus is maximized with the simultaneous regime. The above discussion is summarized in the proposition below. An example is given in the Appendix.

Proposition 5.2 *There exists a range of parameters for which the seller chooses sequential investments when the simultaneous regime maximizes total surplus.*

If the seller opts for the inefficient investment regime the buyer's share of surplus is necessarily reduced. Herein lies how the choice of timing of investment can act as an additional form of hold-up; we term this new form of hold-up 'follow-up'. In the extreme the reduction in surplus may lower the buyer's utility inside the relationship below his outside option: that is $\frac{\delta}{2}[\delta R(\widetilde{I}_1, \widetilde{I}_2) - \widetilde{I}_2] - \widetilde{I}_1 < 0$. In this case the option of the sequential regime prevents trade from occurring;[†] the inability to commit to a particular timing regime (the simultaneous regime) hurts the seller as well as the buyer.[†]

5.6 Predictions of the model

This section analyzes the optimal timing of investment. First, the section explores the relative efficiency of each regime when one investment is very important in terms of its contribution to overall surplus. Second, we examine the situation when one party's investment is invariant to the regime adopted.

5.6.1 Important investments and timing

From effects 5.2 and 5.3 above, sequential investments favor I_2 while simultaneous investments favor I_1. As a consequence, when I_1 is very important relative to I_2, the simultaneous investment system is preferred over sequential investments. Using similar reasoning, when I_2 is very important relative to I_1, the sequential system is favored over the simultaneous investment system, provided both parties are sufficiently patient.

[†]Note that provided $\frac{\delta}{2}R(\widehat{I}_1, \widehat{I}_2) - \widehat{I}_1 > 0$, the buyer would have opted into the relationship if only the simultaneous regime were available.

[†]Of course, the sequential regime can also create trading possibilities not available with only the simultaneous regime. For example, the seller may not be willing to invest simultaneously because the hold-up that occurs during the subsequent renegotiation may leave them with negative utility. On the other hand, sequential investment gives her the opportunity to delay their investment until when contracts are complete. This encourages the seller to invest and allows trade to proceed. This result is similar to the results of other authors, for example Neher (1999) and Admati and Perry (1991).

To see this, we adopt a variant of Hart's (1995) definition of an unimportant investment.† For simplicity we assume $f_1(0) = f_2(0) = 0$.

Definition 5.1 *I_1 is unimportant if $R(I_1, I_2) = \delta f_1(I_1) + \delta f_2(I_2) - I_1 - I_2$ is close to $R(0, I_2) = \delta f_2(I_2) - I_2 \;\forall\; I_1$. Similarly, I_2 is unimportant if $R(I_1, I_2) = \delta f_1(I_1) + \delta f_2(I_2) - I_1 - I_2$ is close to $R(I_1, 0) = \delta f_1(I_1) - I_1 \;\forall\; I_2$.*

The key element here is that when a particular investment is unimportant it contributes relatively little to total surplus, although the marginal incentive to invest for the relevant player is unchanged.† The term 'close to' in Definition 5.1 can be considered as equivalent to the statement that A is close to B iff $A \gg A - B$.

First consider when I_2 is unimportant. Using the definition above, if I_2 is unimportant total surplus with simultaneous investment, $\delta f_1(I_1) + \delta f_2(I_2) - I_1 - I_2$, can be replaced by

$$\delta f_1(I_1) - I_1. \tag{5.18}$$

As a result all that matters to overall welfare is I_1. Surplus is then maximized by the system that promotes the highest level of I_1. As noted above, the level of I_1 with simultaneous investments, $\widehat{I}_1$, is closer to the first-best level than $\widetilde{I}_1$. Following from Definition 5.1:

$$R(\widehat{I}_1, \widehat{I}_2) \approx \delta f(\widehat{I}_1) - \widehat{I}_1 > R(\widetilde{I}_1, \widetilde{I}_2) \approx \delta^2 f(\widetilde{I}_1) - \widetilde{I}_1. \tag{5.19}$$

A similar argument can be made when I_1 is unimportant. In this case the sequential regime provides the seller with greater incentive to invest efficiently. There is, however, additional costs of delay with the sequential regime as compared with the simultaneous regime. The sequential regime will only be preferred if the benefits from the seller's additional investment outweigh these delay costs. From Definition 5.1 the total surplus with simultaneous investments is $\delta f(\widehat{I}_2) - \widehat{I}_2$, whereas the

†See Hart (1995, p. 44).

†The first-order conditions for both players are unchanged from the initial problem. With simultaneous investments $f_i'(I_i) = \frac{2}{\delta}$ for $i = 1, 2$. With sequential investments the first-order condition for the buyer is $f_1'(I_1) = \frac{2}{\delta^2}$ and the seller's first-order condition is $f_2'(I_2) = \frac{1}{\delta}$.

total surplus with sequential investments is given by $\delta^2 f(\widetilde{I}_2) - \delta\widetilde{I}_2$. The following proposition summarizes this discussion.

Proposition 5.3 *When I_2 is unimportant the simultaneous investment system maximizes total welfare. When I_1 is unimportant either regime may maximize total welfare.*

This result parallels Proposition 2(B) in Hart (1995). Hart argued that when one investment was unproductive, asset ownership would be organized as to give the other party as much incentive to invest as possible. The model presented here suggests that when one investment is relatively unimportant, the timing of investment should provide as much incentive as possible to the other party (ignoring the costs of delay). As in Hart (1995) there is no need to worry about the loss of surplus from reducing the other player's investment, because it contributes relatively little to surplus.

5.6.2 Inelastic investments

A party's level of investment may be invariant to the timing regime adopted. This may result, for example, because of a binding wealth constraint. This inelasticity can be utilized by maximizing the incentive for the other party to invest.

To facilitate the discussion consider the following analogue of Definition 1 in Hart (1995, p. 44).

Definition 5.2 *The buyer's (seller's) investment decision is inelastic when his (her) level of investment I_1 (I_2) is the same for both the simultaneous and the sequential regimes.*

If the buyer will invest $\overline{I}_1$ with either regimes, the sequential regime enhances the seller's incentive to invest. There is, however, an additional cost of delay. In terms of maximizing welfare, these two factors work against each other. As a result, either regime could maximize welfare when the buyer's investment is inelastic. Alternatively, when the seller's investment is inelastic - that is, she always invests $\overline{I}_2$

regardless of the regime adopted - the simultaneous regime both encourages greater investment by the buyer and reduces the cost of delay. In this case the simultaneous regime is unambiguously superior. This discussion is summarized in the following proposition.

Proposition 5.4 *When the buyer's investment is inelastic there is an ambiguous relationship between the type of regime and total welfare. If the seller's investment is inelastic, the simultaneous regime unambiguously maximizes total surplus.*

This result is similar to Proposition 2(A) in Hart (1995, p. 45). There, if one party's incentive to invest is invariant to asset ownership the other party should own the assets in order to encourage more efficient investment. Similarly here, when one party's incentive to invest is inelastic to the regime adopted, the regime chosen should maximize the incentive for the other party to invest. The only complication here is that the cost of delay also needs to be taken into account. For example, if the generation of additional surplus from more efficient investment by the seller with the sequential regime does not outweigh the costs of delay, the simultaneous system should still be adopted.

5.7 Extensions

This section makes several extensions to the model presented above. First, we explore the relationship between the two systems when I_1 and I_2 are strategic complement or substitute investments. This allows the relative efficiency of each system to be examined when one player's investment decision is highly sensitive with respect to the regime that is adopted. Second, we investigate the implications for total welfare when there is a lack of commitment so that either party can trigger renegotiation at any point in time.

5.7.1 Strategic substitute and complementary investments

When investment are strategic complements or substitutes $R_{12} \neq 0$. As this can significantly complicate matters, assume that $\delta = 1$, as summarized in Assumption 5.3.

Assumption 5.3 $\delta = 1$

When Assumption 5.3 holds the total surplus is

$$S = R(I_1, I_2) - I_1 - I_2 \tag{5.20}$$

with both regimes. For the two regimes each player will choose their level of investment given their respective first-order conditions, shown in the Appendix. As $R_{12} \neq 0$, an adjustment in one investment will alter the marginal productivity of the other player's investment; this will affect each player's incentive to invest.

If the cross derivative of the investments is positive ($R_{12} > 0$) the investments are strategic complements as an increase in I_1 enhances the marginal productivity of I_2. This is summarized in Definition 5.3.

Definition 5.3 *If $R_{12} > 0$, I_1 and I_2 are strategic complements.*

Complementary investments may arise between trading parties, for example, when investment in a particular location enhances the value of the other party's investment. Effort in learning about the specific requirements of the trading partner can also enhance the productivity of the other player's investment. Similarly, investing in machinery or retooling in such a way to fit the requirements of the trading partner can help increase the marginal product of the other investment.

First, consider the case when investments are simultaneous. As one of the parties shades their investment, this encourages the other party to also shade their investment. The overall effect is that both parties reduce their investments even further below their levels when $R_{12} = 0$. This is the familiar underinvestment of the hold-up literature when there are externalities.[†] As a consequence, when investments are simultaneous and the investments are complementary there is underinvestment in both I_1 and I_2. This is summarized in Result 5.1.

Result 5.1 *When the investments are strategic complements and made simultaneously, there is underinvestment in both I_1 and I_2.*

Now consider when $R_{12} < 0$.

Definition 5.4 *If $R_{12} < 0$, I_1 and I_2 are strategic substitutes.*

An example of strategic substitute investments is when the two parties both require the use of a third asset, such as a particular location or venue, the supply of which is fixed or severely limited.[†] In this case, the buyer using the asset reduces the seller's return on any investment because their use of the asset is subsequently limited.

It is shown in the Appendix that the overall impact on I_1 and I_2 is ambiguous when investments are made simultaneously. For example, if the seller shades her investment the buyer has an incentive to increase I_1. Provided that the substitutability of the investments - as measured by the absolute size of R''_{12} - exceeds the effect of diminishing returns to investment - as measured by the absolute value of R''_{22} - the buyer will have an incentive to increase his investment above the first-best level. Similarly, there can be over-investment in I_2 provided the substitutability of the investments outweighs the negative effect of the diminishing returns of investment ($| R''_{12} |>| R''_{11} |$). It follows from the assumption of concavity, however, that there will be underinvestment in at least one of the investments, even if there is over-investment in one of the investments.[†] The above discussion is summarized in the following result.

[†]For example, see De Fraja (1999).

[†]Another example could be a negative externality between the parties. See, for example, Pitchford and Snyder (1999). Alternatively, if the two parties both produce a byproduct or pollutant, the output of which is limited by government regulation, an increase in output by one party limits the permissible output by the other.

[†]For details see the Appendix.

Result 5.2 *When the investments are substitutes and made simultaneously there can be under or over-investment in I_1 and I_2, however, there will be underinvestment in at least one of the investments.*

Now consider when investments are made sequentially. The buyer will underinvest regardless as to whether the investments are strategic complements or substitutes, as was the case when $R_{12} = 0$. In regards to I_2, when the investments are strategic complements the seller also underinvests.† This is because, unlike when $R_{12} = 0$, the underinvestment in I_1 reduces the incentive for the seller to invest in I_2.

In contrast, when the investments are substitutes, the underinvestment in I_1 by the buyer provides an incentive to the seller to overinvest in I_2 . The following result summarizes the above discussion.

Result 5.3 *When investment is sequential, there is underinvestment in I_1. When investments are strategic complements there is also underinvestment in I_2, while if investments are strategic substitutes there is overinvestment in I_2.*

This subsection has explored the situation when investment by one party affects the marginal productivity of the other's investment, either in a negative or positive manner. It was shown previously that when $R_{12} = 0$ the relative welfare of the two systems depended on the interaction of three effects. When the investments are either complements or substitutes these three effects are complicated somewhat by the impact each investment can have on the other investor's incentives. The next subsection extends this analysis further, notably by relaxing the assumption that $\delta = 1$.

†Note that when $R_{12} = 0$ the sequential regime encouraged the seller to set I_2 at the first-best level.

5.7.2 Substitutes and complements with hyper-incentives

To further explore this issue consider the following specific functional form:

$$R = f_1(I_1) + f_2(I_2) + \varepsilon I_1 I_2 \tag{5.21}$$

such that total surplus with simultaneous investment is

$$\delta R - I_2 - I_1 \tag{5.22}$$

and total surplus with sequential investment is

$$\delta^2 R - I_1 - \delta I_2. \tag{5.23}$$

With this function, when $\varepsilon < 0$ the investments are strategic substitutes and when $\varepsilon > 0$ they are strategic complements.

With the simultaneous regime, the first-order conditions for each party are:

$$\widehat{f_1'} = \frac{2}{\delta} - \varepsilon \widehat{I}_2; \tag{5.24}$$

and

$$\widehat{f_2'} = \frac{2}{\delta} - \varepsilon \widehat{I}_1. \tag{5.25}$$

When investment is sequential the relevant first-order conditions are:

$$\widetilde{f_1'} = \frac{2}{\delta^2} - \varepsilon \widetilde{I}_2; \tag{5.26}$$

and

$$\widetilde{f_2'} = \frac{1}{\delta} - \varepsilon \widetilde{I}_1. \tag{5.27}$$

If $|\varepsilon|$ is small the complementarity or substitutability between I_1 and I_2 will be outweighed by effects 5.1, 5.2 and 5.3, outlined when the investments are independent ($\varepsilon = 0$). As the impact of ε is relatively small it remains the case that $\widehat{I}_1 > \widetilde{I}_1$ and $\widehat{I}_2 < \widetilde{I}_2$, in a similar manner as to when $R_{12} = 0$. Further, there are the same welfare trade-offs between the regimes, namely that simultaneous investment increases the contribution to total welfare from I_1 while the surplus generated by I_2 is enhanced with sequential investment. Note that, however, when $R_{12} \neq 0$ the sequential regime will merely encourage the seller to invest at the surplus maximizing level given the buyer's investment; this will not necessarily be the first-best level. This discussion is summarized in the following remark.

Remark 5.4 *When $R = f_1(I_1) + f_2(I_2) + \varepsilon I_1 I_2$, provided the investments are not strong strategic complements or substitutes, the same three effects outlined in Result 5.1 determine the relative welfare of the simultaneous and sequential regimes. Note, the directions of these three effects remain unchanged, although the values may be different.*

Proof. See the Appendix. □

When $|\varepsilon|$ is large, the effects arising from the interaction between investments can lead to other possibilities. For example, if $\varepsilon > 0$, it is possible for the any one of the relevant first-order conditions to be less than zero. This provides that party with the incentive to invest ∞; given the complementarity between investments, the other party will also invest ∞, and the first-best will be achieved (ignoring the costs of delay). Another interpretation is that the first party will invest as much as they can, given their budget constraint. Again, this will encourage the other party to increase their investment. When a party's derivative is negative, this produces a 'hyper-incentive' for that party to invest. This term is defined below.

Definition 5.5 *A hyper-incentive is created when the first-order condition for a party is negative.*

Interestingly, one regime may produce a negative first-order condition while the other may not. For example, the simultaneous regime may produce a negative first-order condition for the buyer, while the sequential system remains positive. In this case, the simultaneous regime produces a hyper-incentive for the buyer to invest - this means that this regime is favored over the alternative. On the other hand, the sequential regime may produce a hyper-incentive for the seller, while her first-order condition with the simultaneous regime may still be positive. It is not the case that the sequential regime is always preferred, however, as the sequential regime involves additional costs of delay. For sequential investment to be favored these costs of delay must be outweighed by the extra surplus generated from the hyper-incentive. The above discussion is summarized in the following result.

Result 5.4 *When the simultaneous regime creates a hyper-incentive for the buyer it is favored over the sequential regime. When sequential investments generate a hyper-incentive for the seller it is favored over the simultaneous investment regime, provided the players are sufficiently patient.*

Of course, when both systems generate hyper-incentives for a particular party, simultaneous investment is preferred as it avoids some costs of delay.

In this subsection we have relaxed the assumption that $\delta = 1$ when the investments are either complements or substitutes. When the complementarity or substitutability between I_1 and I_2 is sufficiently small the same welfare trade-offs apply as when $R_{12} = 0$: the simultaneous regime encourages investment in I_1 and lowers costs of delay, while the sequential regime encourages investment in I_2. With significant interaction between the investments the matter is further complicated so that other outcomes are possible.

5.7.3 Renegotiation

Hart and Moore (1999) assumed it was not possible for trading parties to make a credible commitment not to renegotiate. Grout (1984) also noted that industrial relations contracts are often not binding. Similarly, in an 'at-will' contracting environment either party can unilaterally trigger renegotiation or terminate the contract if they wish.[†] In this section we assume that either party can trigger renegotiation at any point in time.

[†]See the discussion of 'at-will' contracts in Malcomson (1997). The contracts in this subsection are slightly different from a typical 'at-will' contract environment. Usually in an 'at-will' environment there is an asymmetry in the bargaining power between the buyer and the seller. For

When this is the case, only the final renegotiation affects the distribution of surplus (and hence the incentive to invest). The last opportunity to renegotiate occurs after the last investment has been made, that is, once I_2 has been completed. Renegotiation will always occur at this stage because the buyer is better off with a new distribution of surplus after I_2 is sunk.

First, consider when investment is simultaneous. As before, renegotiation will occur after both investments have been made. Consequently, the first-order conditions for both players are the same as described above. With sequential investment renegotiation will always occur after the seller has invested. As both investments are sunk, the parties will split the surplus 50-50. The buyer will set his investment to maximize:

$$\frac{\delta^2}{2}[R(I_1, I_2)] - I_1. \tag{5.28}$$

His first-order condition under these circumstances will be

$$R_1' = \frac{2}{\delta^2} \tag{5.29}$$

which is unchanged from when there is no subsequent renegotiation. Label the level of the buyer's investment when commitment is not possible with sequential investment as $\tilde{\tilde{I}}_1$. From this it can be seen that

$$\tilde{\tilde{I}}_1 = \tilde{I}_1 < \hat{I}_1. \tag{5.30}$$

On the other hand, the seller will maximize:

$$\frac{\delta^2}{2}[R(I_1, I_2)] - \delta I_2 \tag{5.31}$$

---

example, if the buyer (firm) starts negotiations and proposes a new lower price, the seller (worker) is taken to have accepted this new proposed contract if she continues to supply her services (labor). On the other hand, if the seller attempts to raise price, the default price takes precedence, unless the buyer explicitly accepts the new contract.

which yields the first-order condition

$$R_2' = \frac{2}{\delta}. \tag{5.32}$$

Label the seller's choice of her investment when commitment is not possible at any stage and investment is sequential as $\tilde{\tilde{I}}_2$. Comparing the first-order condition for the seller when there is simultaneous investment ($R_2' = \frac{2}{\delta}$), and when investment is sequential but there is no after-investment renegotiation ($R_2' = \frac{1}{\delta}$), it can be seen that

$$\tilde{\tilde{I}}_2 = \widehat{I}_2 < \widetilde{I}_2. \tag{5.33}$$

If there is ex post renegotiation it does not matter that the investments were initially made sequentially as both parties suffer from hold-up. As the buyer is always held-up, assuming $R_{12}'' = 0$, his incentive to invest is unchanged from the usual sequential regime discussed above. Now, however, any potential advantage of the sequential regime is eliminated: the seller also suffers from hold-up with the sequential regime reducing her incentive to invest. As the sequential system involves more costs of delay, the simultaneous system produces higher total surplus than the sequential regime. Consequently, if commitment is not possible, simultaneous investment is strictly preferred to sequential investment. Moreover the ability of either party to trigger renegotiation at any time effectively renders the possibility of sequential investment (or its attractiveness) redundant. This is summarized in the following result.

Result 5.5 *If the parties cannot commit not to renegotiate after both investments have been made, the simultaneous system strictly dominates sequential investment for $\delta < 1$ in terms of total welfare as well as welfare of the seller. Consequently, if the parties are unable to commit not to renegotiate, the sequential regime is never adopted.*

This lack of commitment may be advantageous, however, if the seller would like to commit not to adopt the sequential regime, as it provides the buyer with a lower level of surplus than his outside option (as discussed in section 5.5). The knowledge that the buyer will trigger renegotiation acts as a credible commitment by the seller to invest simultaneously. This may in turn encourage the buyer to invest.

5.8 Conclusion

This chapter develops a model in which two parties can invest in a mutually beneficial project at the same time (simultaneous investment) or they can choose to invest one after the other (sequential investment). It is assumed that contracting on any future investment becomes possible after some investment has been made as it allows the project to become more clearly defined. Consequently, the advantage of the sequencing of investments is it allows the party that has delayed making their investment to avoid being held-up. The disadvantage of staging is that it reduces the incentive to invest of the first-mover. This can also have feed-back effects on the second party's investment depending on the relationship between the two investments. In addition, sequencing of investment lengthens the time from the start of the project until the returns are realized, reducing the ex ante value of total surplus when parties discount future returns. The relative advantage of the sequential versus the simultaneous investment regime depends on the precise nature of these trade-offs. Two principles apply, however, provided the parties are sufficiently patient: first, the regime that favors the most important investment in terms of its contribution to total surplus is preferred; and, second, if one investment is invariant to the regime adopted, the optimal timing of investment will be the regime that maximizes the incentive for the other party to invest.

Much of the emphasis in the existing literature has focused on how staging investments can improve welfare when there are incomplete contracts or when parties are unable to commit. In the model presented in this chapter it is demonstrated that, in some cases, the option of sequencing investments can reduce welfare. It is shown that under certain conditions a party will opportunistically opt for the sequential regime, reducing total surplus. We interpret this possibility as a new form of hold-up and term it 'follow-up'.

Appendix

Example 5.1 *Consider the case when $R(I_1, I_2) = \alpha ln I_1 + \beta ln I_2$. Figure 5.3 shows the four different surpluses for both simultaneous and sequential investments when contracts are both complete and incomplete.*[†] *First note that S^*, the total surplus when investment is contractible and simultaneous, and S^{**}, the total surplus when both investments are contractible but made sequentially, are equal when $\delta = 1$ as there are no costs of delay. Second, consider the surplus generated when contracts are incomplete. S_{sim} represents the total surplus with simultaneous investment, while S_{seq} represents the total surplus with the sequential regime. With low values of δ, S_{sim} exceeds S_{seq}. However, for values of δ greater than about 0.9, $S_{seq} > S_{sim}$; that is, the total surplus from sequential investments exceeds the total surplus with the simultaneous regime.*

Example 5.2 *As an example consider the following explicit function where: $f_1 = aI_1^e$ and $f_2 = bI_2^c$. Here, consider the case when $a = 11$, $b = 10$, $c = 0.3$ and $e = 0.7$. Using the explicit solutions to each party's first-order condition, the total utility generated with simultaneous investment can be written as a function of δ: $S_{sim}(\delta) = a\delta\left(\frac{ea\delta}{2}\right)^{\frac{e}{1-e}} - \left(\frac{ea\delta}{2}\right)^{\frac{1}{1-e}} + b\delta\left(\frac{cb\delta}{2}\right)^{\frac{c}{1-c}} - \left(\frac{cb\delta}{2}\right)^{\frac{1}{1-c}}$. Similarly, the total surplus with sequential investment is: $S_{seq}(\delta) = a\delta^2\left(\frac{ea\delta^2}{2}\right)^{\frac{e}{1-e}} - \left(\frac{ea\delta^2}{2}\right)^{\frac{1}{1-e}} + b\delta^2(cb\delta)^{\frac{c}{1-c}} - \delta(cb\delta)^{\frac{1}{1-c}}$. Figure 5.4 compares these two surpluses. First, there is clearly a non-monotonic relationship between δ and the difference between $S_{sim}(\delta)$ and $S_{seq}(\delta)$. Second, the two functions cross twice, once when δ is close to 0 and another time when δ is close to 1.*

[†] *With simultaneous investment and complete contracts the first-order conditions are $\frac{I_1}{\alpha} = \frac{I_2}{\beta} = 1/\delta$. When contracts are incomplete and investments are simultaneous the first-order conditions are $\frac{I_1}{\alpha} = \frac{I_2}{\beta} = 2/\delta$. When investments are sequential and contracts complete: $\frac{I_1}{\alpha} = 1/\delta^2$ and $\frac{I_2}{\beta} = 1/\delta$. Finally, when investments are sequential and contracts incomplete the first-order conditions are: $\frac{I_1}{\alpha} = 2/\delta^2$ and $\frac{I_2}{\beta} = 1/\delta$. The specific functions used assume $\alpha = \beta = 5$: that is $S^*(\delta) = 10\delta(\ln 5\delta - 1)$, $S_{sim}(\delta) = 10\delta(\ln 5\delta - 0.5 - \ln 2)$, $S^{**}(\delta) = 5\delta^2(\ln 5\delta^2 - 1) + 5\delta^2(\ln 5\delta - 1)$ and $S_{seq}(\delta) = 5\delta^2(\ln 5\delta^2 - 0.5 - \ln 2) + 5\delta^2(\ln 5\delta - 1)$.*

Example 5.3 *Consider the case when* $R = 10lnI_1 + 8lnI_2$. *Figure 5.5 plots the surplus of the seller with different investment regimes (on the Y-axis) against* δ *(on the X-axis).* S^s_{sim} *shows two times the seller's surplus when investments are made simultaneously.* S_{seq} *shows two times the surplus of the seller - this equals the total surplus - when investments are made sequentially.* S_{sim} *shows the total surplus of both parties with simultaneous investments. It can be seen that for* $\delta > 0.8$ *(approximately) the seller will opt for the sequential system over the simultaneous option. However, from* S_{sim} *and* S_{seq} *it is only when* $\delta > 0.95$ *(approximately) that the sequential system produces more surplus than simultaneous regime. Thus, for* $\delta \in (0.8, 0.95)$ *the seller opts for the regime that does not maximize total welfare. Also note, in this example the buyer's investment is assumed fixed at* $\widehat{I}_1$ *for all of the functions. The specific functions used are* $S_{sim}(\delta) = \delta(10\ln 5\delta + 8\ln 4\delta) - 4\delta$, $S^s_{sim}(\delta) = \delta(10\ln 5\delta + 8\ln 4\delta) - 8\delta$ *and* $S_{seq}(\delta) = \delta^2(10\ln 5\delta + 8\ln 8\delta) - 8\delta$.

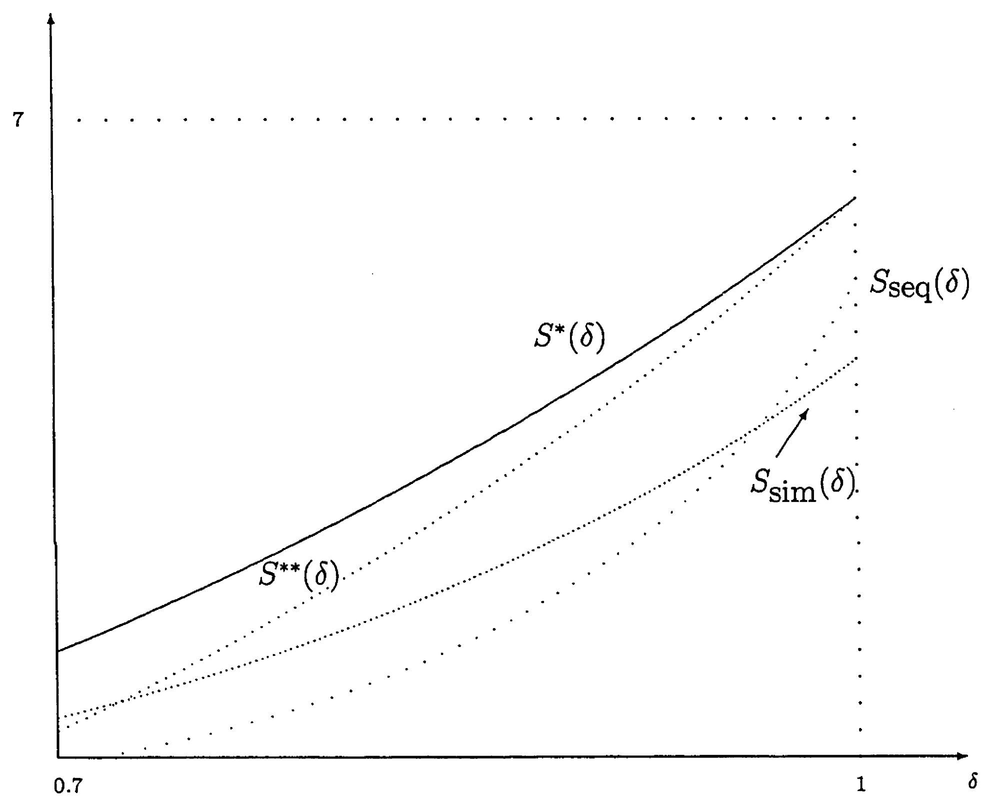

Figure 5.3: Illustration to example 5.1

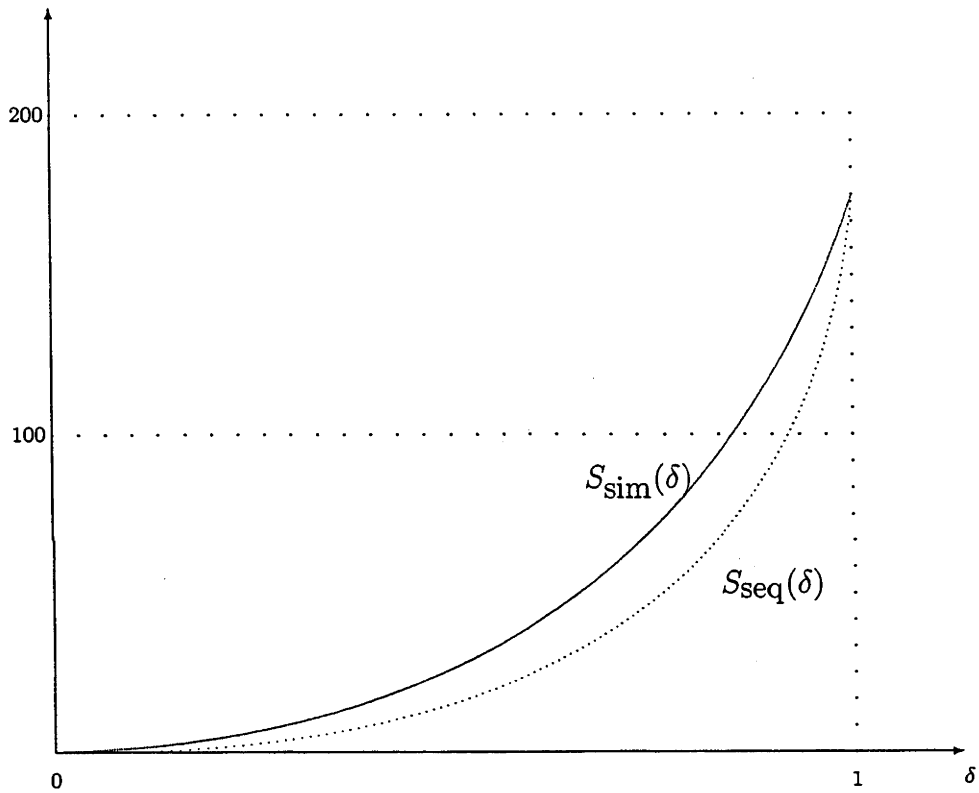

Figure 5.4: Illustration to example 5.2

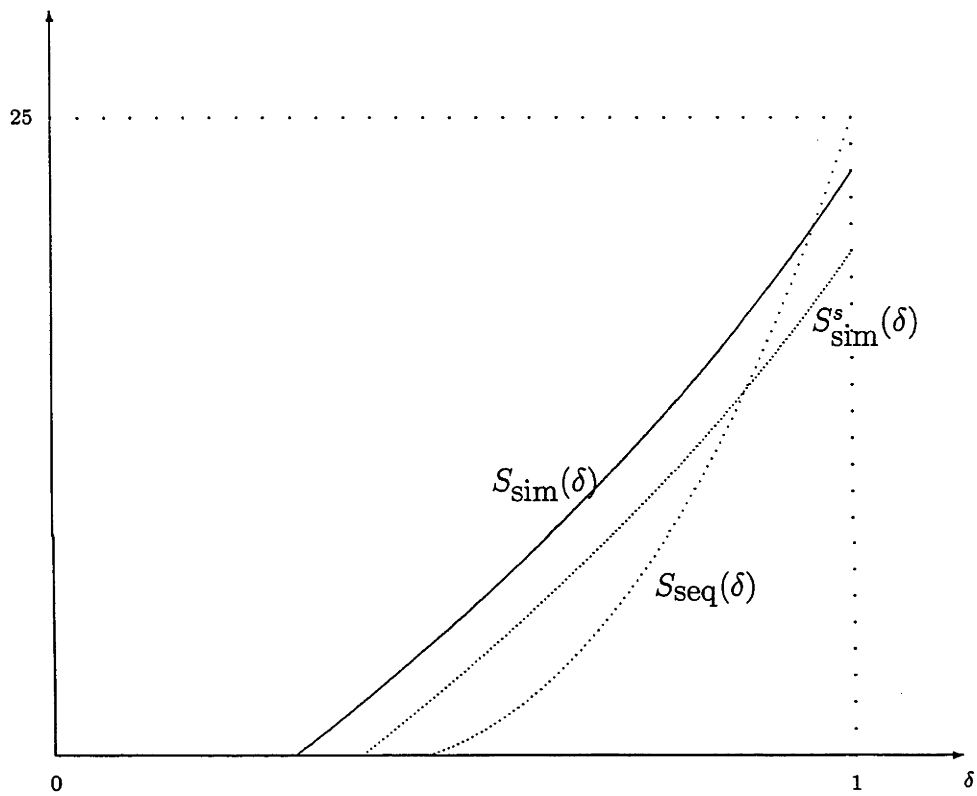

Figure 5.5: Illustration to example 5.3

Result 5.1 *When the investments are complements and made simultaneously, there is underinvestment in both I_1 and I_2.*

Proof. When Assumption 5.3 holds the total surplus is

$$S = R(I_1, I_2) - I_1 - I_2 \tag{5.34}$$

for the levels of investment chosen in the different systems. The first-order conditions are

$$\widehat{R}_1' = 2 \tag{5.35}$$

$$\widehat{R}_2' = 2 \tag{5.36}$$

for the simultaneous investment system, and

$$\widetilde{R}_1' = 2 \tag{5.37}$$

$$\widetilde{R}_1' = 1 \tag{5.38}$$

with sequential investments.

To investigate this further, replace substitute $a \in [1, 2]$ for 2 in each of the equations, so that

$$\widehat{R}_1' = a \tag{5.39}$$

$$\widehat{R}_2' = a \tag{5.40}$$

for the simultaneous investment equations, and

$$\widetilde{R}_1' = a \tag{5.41}$$

$$\widetilde{R}_1' = 1 \tag{5.42}$$

for the sequential system. This allows the buyer and seller's investment levels to be represented as functions of a: from equations 5.39 and 5.40 the relevant investment levels become $\widehat{I}_1(a)$ and $\widehat{I}_2(a)$; and from equations 5.41 and 5.42 $\widetilde{I}_1(a)$ and $\widetilde{I}_2(a)$ are the relevant investment levels. Totally differentiating equations 5.39 and 5.40 with respect to a yields

$$R_{11}^{''}I_1^{'}(a) + R_{12}^{''}I_2^{'}(a) = 1 \tag{5.43}$$

$$R_{21}^{''}I_1^{'}(a) + R_{22}^{''}I_2^{'}(a) = 1. \tag{5.44}$$

Solving this system of equations using Cramer's rule yields solutions

$$\widehat{I_1^{'}}(a) = \frac{R_{22}^{''} - R_{12}^{''}}{R_{11}^{''}R_{22}^{''} - (R_{12}^{''})^2} \tag{5.45}$$

$$\widehat{I_2^{'}}(a) = \frac{R_{11}^{''} - R_{12}^{''}}{R_{11}^{''}R_{22}^{''} - (R_{12}^{''})^2}. \tag{5.46}$$

Note that given the assumption of concavity the denominator is always negative.

When $R_{12} > 0$,

$$\widehat{I_1^{'}}(a) < 0 \tag{5.47}$$

$$\widehat{I_2^{'}}(a) < 0. \tag{5.48}$$

The overall effect of moving from the first-best level of investment (when $R_i^{'} = 1$) to the second best solutions given by equations 5.35 and 5.36, must consider the integral of the marginal changes over the entire range of $a \in [1, 2]$. However, as the marginal change is always of the same sign we can discern that when the investments are complements there is underinvestment of both investments. □

Result 5.2 *When the investments are substitutes and made simultaneously there can be under or overinvestment in I_1 and I_2, however, there will be underinvestment in at least one of the investments.*

Proof. From equations 5.45 and 5.46, when $R_{12} < 0$,

$$\widehat{I_1^{'}}(a) \gtrless 0 \tag{5.49}$$

$$\widehat{I_2^{'}}(a) \gtrless 0. \tag{5.50}$$

For I_1, the derivative is positive if $\mid R_{12}^{''} \mid > \mid R_{22}^{''} \mid$. Likewise, the derivative for

I_2 is positive if $\mid R''_{12} \mid > \mid R''_{11} \mid$. In addition, it follows from the assumption of concavity that $I'_1(a) + I'_2(a) = \frac{R''_{22}+R''_{11}-2R''_{12}}{R''_{11}R''_{22}-(R''_{12})^2} < 0$. This suggests that there will be underinvestment in at least one of the investments, even if there is over-investment in one of the investments. □

Result 5.3 *When investment is sequential, there is underinvestment in I_1. When investments are complements there is also underinvestment inI_2 while if investments are substitutes there is overinvestment in I_2.*

Proof. As above, totally differentiating the equations 5.41 and 5.42 yields

$$R''_{11}I'_1(a) + R''_{12}I'_2(a) = 1 \tag{5.51}$$

$$R''_{21}I'_1(a) + R''_{22}I'_2(a) = 0. \tag{5.52}$$

Solving using Cramer's rule shows that

$$\tilde{I}'_1(a) = \frac{R''_{22}}{R''_{11}R''_{22} - (R''_{12})^2} \tag{5.53}$$

$$\tilde{I}'_2(a) = \frac{-R''_{12}}{R''_{11}R''_{22} - (R''_{12})^2}. \tag{5.54}$$

Regardless of the sign of R_{12},

$$\tilde{I}'_1 < 0. \tag{5.55}$$

This indicates that there will be underinvestment in I_1.

For I_2, when the investments are complements - that is when $R''_{12} > 0$ - there is underinvestment in I_2 as

$$\tilde{I}'_2 < 0. \tag{5.56}$$

When $R''_{12} < 0$,

$$\tilde{I}'_2 > 0 \tag{5.57}$$

indicating that there will be over-investment in I_2. □

Remark 5.4 *When $R = f_1(I_1) + f_2(I_2) + \varepsilon I_1 I_2$, provided the investments are not strong complements or substitutes the same three effects outlined in section 5.3 determine the relative welfare of the simultaneous and sequential regimes. Note, the directions of these three effects remain unchanged, although the values may be different.*

Proof. Let us consider the following parameterized first-order conditions

$$f_1'(I_1) = a - \varepsilon I_2 \text{ and } f_2'(I_2) = b - \varepsilon I_1. \tag{5.58}$$

The following equation on the optimal level of I_1 can be derived from the above system:

$$f_2'\left(\frac{a - f_1'(I_1)}{\varepsilon}\right) = b - \varepsilon I_1. \tag{5.59}$$

Differentiating this equation with respect to I_1 when b = constant and $a = a(I_1)$ gives

$$f_2''(\cdot)\frac{a' - f_1''(\cdot)}{\varepsilon} = -\varepsilon, \tag{5.60}$$

which means

$$\frac{\partial I_1}{\partial a} = \frac{1}{a'} = \frac{f_2''(\cdot)}{f_2''(\cdot)f_1''(\cdot) - \varepsilon^2} < 0. \tag{5.61}$$

Similarly when a = constant and $b = b(I_1)$ differentiating equation 5.59 with respect to I_1 gives

$$f_2''(\cdot)\frac{-f_1''(\cdot)}{\varepsilon} = b' - \varepsilon, \tag{5.62}$$

from which it follows

$$\frac{\partial I_1}{\partial b} = \frac{1}{b'} = \frac{\varepsilon}{\varepsilon^2 - f_2''(\cdot)f_1''(\cdot)} > 0. \tag{5.63}$$

When $|\varepsilon|$ is small the effect outlined in equation 5.63 can be ignored. Consequently, equation 5.61 has the dominant effect. From this we know that I_1 is higher with simultaneous investment than with the sequential regime, and that I_1 is greater still with complete contracts (first-best I_1). Further, in a similar manner as outlined in Lemma 5.1, higher levels of I_1 translate to a greater contribution to total surplus. We can rank the regimes in terms of the contribution I_1 makes to welfare: the simultaneous regime dominates the sequential regime.

We now derive the equation on the optimal level of I_2 from the parameterized system

$$f_1'\left(\frac{b - f_2'(I_1)}{\varepsilon}\right) = a - \varepsilon I_2. \tag{5.64}$$

Differentiating this equation with respect to I_1 when: $b =$ constant and $a = a(I_1)$; and when $a =$ constant and $b = b(I_1)$ gives

$$\frac{\partial I_1}{\partial a} = \frac{1}{a'} = \frac{\varepsilon}{\varepsilon^2 - f_2''(\cdot)f_1''(\cdot)} > 0 \tag{5.65}$$

and

$$\frac{\partial I_1}{\partial b} = \frac{1}{b'} = \frac{f_1''(\cdot)}{f_2''(\cdot)f_1''(\cdot) - \varepsilon^2} < 0 \tag{5.66}$$

respectively.

In a similar manner as described with I_1 above, when $|\varepsilon|$ is small the effect outlined in equation 5.66 has the dominant influence on I_2. This suggests I_2 is greater with the sequential regime than with simultaneous investments, although it is still lower than its first-best level. The levels of I_2 also directly translate into its contribution to total welfare: I_2 contributes more to total welfare with sequential investment than with the simultaneous regime. □

CHAPTER 6

Industry sunk costs and entry dynamics

6.1 Introduction

The development of a new market often involves sunk costs. For example, a firm may need to invest heavily in advertising in order to generate knowledge and stimulate interest in a new product. Importantly, it can be the case that a significant component of these costs are industry sunk costs, as opposed to firm-specific sunk costs. Similarly, investment in research and development can aid potential competitors when intellectual property rights are poorly protected possibly internationally.[†] This paper incorporates industry sunk costs into a strategic model of investing as a leader or as a follower.

The basics of the model are as follows. Before any firm can exploit a new profitable market opportunity, a certain amount of resources need to be expended on either advertising, to inform the public of the new product, or on non-patented research. This cost is borne by firms that initially enter the market, but this expenditure is a public good for all potential entrants in that, once the investment has been sunk, all firms can benefit from this investment if they choose to enter the market. The question then arises for each firm as to when they should enter the market: early entry allows them to benefit with fewer competitors but may mean they incur some of the industry set-up costs; delayed entry may allow a firm to avoid the set-up costs, but they also forgo some benefits by not participating in the

[†]For example, see Stegemann (2000), Ostergard (2000) and Levy (2000) for a discussion of international protection of intellectual property and copyright. In another context, Roberts (2000) argued that patents provided limited protection to internet companies and their technologies.

market.

Several interesting results arise out of the model. First, consider the case when there are two potential entry periods. This means a firm can enter immediately or it can sit out of the market for one period and enter in the next period. A firm cannot enter the market if it decided not to enter in either of the first two periods. If sunk costs are sufficiently high, each firm has a dominant strategy to wait and not enter the market until the second potential investment period. This result is a type of prisoners' dilemma: welfare is reduced by the delay in entry but no firm has an incentive to deviate.

Second, as the number of potential investment periods are increased (from two periods), the benefit of waiting at the start of the game is reduced as future returns are discounted. With a potential horizon that is sufficiently long, the firms will adopt a mixed strategy between investing and not investing in this period: this is a coordination game.[†]

Third, interesting dynamics can arise as the potential investment horizon is further extended, in which it is possible for the outcome of the game to switch between a prisoners' dilemma and a coordination game. For example, consider the case when the firms play a mixed strategy when there are k periods. With an additional potential investment period, in the first of the $k+1$ periods each firm will consider the benefit of not investing immediately: this is the outcome of the k-period coordination game. This outcome may be sufficiently large to provide an incentive for the firms to wait; the game has reverted to a prisoners' dilemma. This generalizes the result of Chapter 4 to n players.

Fourth, in the infinite-horizon game firms play a mixed strategy in the unique symmetric equilibrium. Of course, other non-symmetric equilibria also exist.

Last, once one player has entered the market, all other potential suppliers enter as soon as possible. This creates an entry cascade. A similar result was generated by Zhang (1997) when firms have differing private information regarding an investment

[†]Note, this game differs from the usual coordination game somewhat: it is, instead, similar to what Binmore (1992) described as an Australian Battle of the Sexes.

opportunity. In Zhang's model after a strategic delay, the best informed firm enters the market, and following this all other firms enter immediately in an investment cascade. The result in the model presented here is significant as it generates an investment cascade without private information.

6.2 n-player investment game

Consider the following set-up. There are $n \geq 2$ firms that are potential entrants to a some new market. The net benefit from entering is B per period, to be shared amongst all firms that have entered.[†] All parties discount future returns by δ per period. There are some costs C that are incurred in the first period in which entry occurs, where C is shared among all the firms that enter in that initial period. Entry (by at least one firm) is efficient in that $\frac{B}{1-\delta} > C$.[†]

First, consider the situation when there are only two potential entry periods, so that a firm can enter in the first period, enter in the second period, or decide to not enter the market at all.[†] Let us show that it is always profitable for a firm to enter the market in the second period, if it has not already done so. Consider the situation when no firm entered the market in the first period. If m firms enter in the second period, the payoff to any individual firm from entering, evaluated at the time, is $\frac{\delta B}{m(1-\delta)} - \delta\frac{C}{m}$. As $\frac{B}{(1-\delta)} > C$ entry is profitable for every firm. If at least one firm entered in the first period the payoff to a firm from entering in the second period, again evaluated at the time, is $\frac{\delta B}{m(1-\delta)}$ if a total of m firms entered over both periods. Clearly entry is profitable in this case. As a consequence, if a firm has not already done so, it will enter the market in the final potential investment period.

Second, using the result above, now consider a firm's decision as to whether or not to enter the market in the first period. To examine this issue we consider: (a)

[†] B could represent profits in the industry that the firms share with perfect collusion, as assumed by Mankiw and Whinston (1986).

[†] Note that, given there are no firm specific sunk costs, the welfare outcome in this model is the same regardless of the number of firms producing, provided at least one firm is in the market.

[†] This two-period investment game has a similar structure to the bank run game analyzed by Gibbons (1992, pp. 73-75) and Diamond and Dybvig (1983) and Chamley's (2001) model of exchange rate speculation.

the payoffs from entry when the firm is the only entrant; and (b) when they share entry in the first period.

If $n-1$ firms decide to wait, the benefit to the other firm from entering in the first period is

$$\frac{B}{n(1-\delta)} + \left(1 - \frac{1}{n}\right) B - C. \tag{6.1}$$

If the firm does not enter in the first period, all of the other firms will enter in the second period, so the payoff to this firm is just discounted by one period payoff of all firms entering

$$\frac{\delta B}{n(1-\delta)} - \frac{\delta C}{n}. \tag{6.2}$$

Payoff of waiting (not investing in the first period) is bigger if and only if

$$(1 - \frac{\delta}{n})C > B. \tag{6.3}$$

Conversely, the firm will enter in the first period, given all the other firms do not enter, if $B > (1 - \frac{\delta}{n})C$.

Now consider the entry decision of one firm when $k > 0$ of the other firms decide to enter and $n-k-1$ decide to wait. If the firm enters it will get the following benefit

$$\frac{B}{n(1-\delta)} + \left(\frac{1}{k+1} - \frac{1}{n}\right) B - \frac{C}{k+1}. \tag{6.4}$$

On the other hand, if it does not enter it will get

$$\frac{\delta B}{n(1-\delta)}. \tag{6.5}$$

Comparing these two equations one can infer that the benefit of waiting is bigger iff

$$C > B. \tag{6.6}$$

From both these cases, the firm has a dominant strategy to invest immediately if $B > C$. When $C > B > C(1 - \frac{\delta}{n})$, the firm prefers to wait if k other firms enter but invest if no other firms invest. Consequently, the firm will adopt a mixed strategy.

Finally, if $B < C(1 - \frac{\delta}{n})$, the firm has a dominant strategy to wait and not invest in the first period.

This discussion is summarized in Proposition 6.1.

Proposition 6.1 *If $B > C$ all firms invest immediately in the first period in any subgame perfect equilibrium (SPE). If $C > B > C(1-\frac{\delta}{n})$ each firm will mix between entering immediately and waiting to enter in the second period in the SPE. Finally, if $B < C(1 - \frac{\delta}{n})$, in the SPE all firms will wait and only enter the market in the second period.*

If $B < C(1 - \frac{\delta}{n})$ the firms are in a prisoners' dilemma: the welfare of every firm would be improved if they all could commit to invest immediately, but here each firm has a dominant strategy to wait, reducing total surplus.

Assume that $B < C(1-\frac{\delta}{n})$, so that the players are in a prisoners' dilemma in the two-period investment game. Now consider the optimal strategies of the firms when there are three potential investment periods. In this case, the payoff from waiting for a firm if no one invests in the first period is the two-period payoff discounted by an additional δ - the extra period of delay reduces the benefit of waiting. Reducing the benefit from waiting makes immediate entry more attractive. If this reduction in the benefit from waiting is sufficient, a firm will no longer have a dominant strategy to wait. Instead they will adopt a mixed strategy between investing and waiting. Note, the mixed strategy equilibrium is a coordination game similar in structure to what Binmore (1992) described as the Australian Battle of the Sexes. In this coordination game, players wish to coordinate to undertake the activity that the other players do not do. Alternatively, of course, the reduction in the benefit from waiting will not be sufficient, and the players will remain in a prisoners' dilemma entry game.

6.3 Switching between prisoners' dilemma and the coordination game

In the previous section we implicitly assumed that the welfare outcome in the model is the same regardless of the number of firms producing, provided at least one firm is in the market.† If this is the case once the game switches from a prisoners' dilemma to a coordination game, the players optimal strategies will remain a coordination game in every subsequent period as the number of periods is increased.† However, if we generalize this model to allow firm specific sunk costs then the equilibrium strategies of the players can switch between prisoners' dilemma and a coordination game as the number of potential investment periods is increased.

Now consider the following generalization of the basic model. Let R_k is the payoff of a player if he invests and Q_k is the payoff if he does not invest; k indicates number of players investing. We assume that

$$Q_{n-1} \geq \ldots Q_1 \geq R_n \geq \ldots \geq R_1. \tag{6.7}$$

This assumption states that benefits from investing second are higher when more players invest first. As being a follower rather than a leader is associated with sharing some additional positive surplus, the less players that invest second, the larger is the share of surplus received by every second period investor. Using a similar logic, one can infer that costs from investing first are higher when less players invest first, because less players share the entrance costs.

As we mentioned above, in this generalized set-up the equilibrium strategies of the players can switch between prisoners' dilemma and a coordination game, as the number of potential investment periods is increased. As an illustration of this consider the following example for the three player game.

†If $0 < k \leq n$ players invest in the first period and the rest $n - k$ players invest in the second period, the welfare outcome is equal to the joint benefits minus the joint costs, namely $B/(1-\delta)-C$. This outcome does not depend on k, while its distribution among players alters with k.

†The proof for this result is contained in the Appendix.

Example 6.1 *This example shows the possibility of switching between a prisoners' dilemma and a coordination game when there are many potential investment periods.*

Let $\delta = 0.9$, $Q_2 = Q_1 = 8.5$, $R_3 = 25/3$, $R_2 = 8$ and $R_1 = 6.5$. Figure 6.1 illustrates the normal form game of the investment decision for three parties when there are $k = 2$ potential investment periods. Note, that the payoff of players when they invest in the second period is discounted payoff of investing in the first period, namely $Q_0 = R_3 * \delta^{k-1}$.†

Player 3 - I

Player 1 \ Player 2	I	W
I	$\frac{25}{3}, \frac{25}{3}, \frac{25}{3}$	$8, 8.5, 8$
W	$8.5, 8, 8$	$8.5, 8.5, 6.5$

Player 3 - W

Player 1 \ Player 2	I	W
I	$8, 8, 8.5$	$6.5, 8.5, 8.5$
W	$8.5, 6.5, 8.5$	$7.5, 7.5, 7.5$

Figure 6.1: A three player strategic game, in which player 3 chooses I or W.

If $k = 2$ and 3, the SPE strategy of both players is to wait - this is a prisoners' dilemma. When $k = 4$, the payoffs for (W, W, W) change from 7.5 to 6.075. This payoff is now less than the payoff of the player if he invests and others do not, namely 6.5. Consequently, there is no dominant strategy available in the first potential investment period when $k = 4$. This is a coordination game. The discounted mixed payoff from the above-mentioned coordination game is 6.534, which is higher than 6.5. For $k = 5$ the game reverts to a prisoners' dilemma. □

6.4 Infinite horizon game

Consider when n firms face a potentially infinite-horizon. Here, every firm can choose to invest immediately (I) or can choose to wait (W). There are $n+1$ stationary SPE in this game. The first involves players $2, 3 \dots n-1$ playing the following strategy: do not invest in the first period; do not invest in the second period unless

†For more detail see the investment game in Chapter 4.

player 1 invested in period $t = 1$; do not invest in the third period unless player 1 invested in period $t = 2$; and so on. Player 1 plays the following strategy: invest in the first period; if not in the first, invest in the second; if not in the second, invest in the third; and so on. In this equilibrium player 1 invests immediately and players $2, 3 \ldots n-1$ follow up with their investment in the next period. Neither player has an incentive to deviate in any subgame. Players $2, 3 \ldots n-1$ receive the highest payoff possible in this game - Q_{n-1} - while player 1 receives a payoff of R_1. If player 1 deviates to invest in the second period, she will receive a payoff of δR_1, ruling out any possibility of a profitable deviation. Symmetrically equivalent equilibria exist in which some other player invests immediately and remaining players invest in the second period.

A stationary mixed strategy equilibrium also exists. In this equilibrium all parties invest with some positive probability. For example, player 1 invests immediately with probability α_1, player 2 invests immediately with probability α_2, and so on. If at least one player invests, the entire investment process will last no longer than two periods and the game will end. However, if all parties do not invest in the first period, which occurs with probability $(1-\alpha_1)(1-\alpha_2)\ldots(1-\alpha_n)$, the players return to an identical situation only one period in the future. In this continuation game the players will again adopt the same strategies. As a result the expected payoff of each player are exactly the same as at $t = 1$, however, they are discounted from the delay of one period. The following proposition states that in the infinite-period case symmetric stationary SPE always exists and is unique.

Proposition 6.2 *In the infinite horizon investment game always exists a unique symmetric mixed strategy SPE.*

Proof. See the Appendix. □

6.5 Cascading investment

Finally, consider the observable entry behavior of firms in either the finite or infinite-horizon game when firms are either mixing between immediate entry and waiting. In this case there is a positive probability that no entry occurs for one or more periods. However, once at least one firm has entered, all firms enter at the next possible opportunity, as in an investment cascade. A similar dynamic could exist in the finite-horizon model where the optimal strategies of the firms switch between having a dominant strategy to wait (prisoners' dilemma) and mixing (the coordination game). Again, the industry may display no entry for several periods, but once entry has occurred all firms immediately enter. This is summarized in Proposition 6.3.

Proposition 6.3 *An investment cascade can result in an investment game when there are industry sunk costs.*

Also note, if the firms have a finite number of potential investment periods and they have a dominant strategy to wait until the last period to enter, the entry behavior of the firms looks similar to the investment cascade: there is no entry for $n - 1$ periods after which time all firms entry in the n-th period.

6.6 Conclusion

This chapter considers the choice between being a leader or a follower in an investment game, where there is a number of potential investors in a new profitable market opportunity. They could either invest in the first period and incur some entrance costs or they could wait and invest in the second period. The following interesting results arise out of the model. First, the switching result of Chapter 4 is generalized to n players. Namely, it is possible that with two potential investment periods the parties find themselves in a prisoners' dilemma; with three potential investment periods they mix between investing immediately and waiting; and with four potential investment periods the players again return to a prisoners' dilemma. Second, the model can display an entry cascade, quite often observed in empirical studies of technology diffusion. Once one player has entered the market, all other potential suppliers enter as soon as possible.

Appendix

Proposition 6.2 *In the infinite horizon investment game always exists a unique symmetric mixed strategy SPE.*

Proof. First, let us prove that there is at least one symmetric mixed strategy SPE. The following condition is assumed.

$$Q_{n-1} \geq \ldots Q_1 \geq R_n \geq \ldots \geq R_1 \geq Q_0. \tag{6.8}$$

Let us find the mixed strategy equilibrium. The probability of investing p solves the following equation

$$\begin{aligned}(1-p)^{n-1}R_1 + Q_{n-1}^1 p(1-p)^{n-2}R_2 + \ldots + p^{n-1}R_n = \\ (1-p)^{n-1}Q_0 + Q_{n-1}^1 p(1-p)^{n-2}Q_1 + \ldots + p^{n-1}Q_{n-1}.\end{aligned} \tag{6.9}$$

The mixed strategy payoff is either the left hand side or right hand side of this equation, and this payoff satisfies the following condition

$$d * payoff = Q_0. \tag{6.10}$$

Thus, our goal is to find such Q_0 that equations 6.9 and 6.10 are satisfied. Note, that if we assume $Q_0 = R_1$, then from equation 6.9 it follows $p = 0$ and $payoff = R_1$, which means $d * payoff < Q_0$. On the other hand, when $Q_0 = 0$, it follows that $d * payoff > Q_0$ (the payoff is strictly positive by definition). Thus, there is always at least one value of Q_0 that satisfies both equations.

Now, let us prove that this value is always unique. First, let us show that there is always only one p that solves equation 6.9 for any given parameter values $Q_0, Q_1 \ldots Q_{n-1}, R_1, R-2 \ldots R_n$. Specifically, if we divide this equation by $(1-p)^{n-1}$ and denote $q := p/1-p$, we get

$$Q_{n-1}^1(Q-1-R_2)q + \ldots + (Q_{n-1} - R_n)q^{n-1} = Q_0 - R_1. \tag{6.11}$$

The value in the right hand side of this equation, and all the coefficients in the left hand side, are positive constants. It means that the left hand side is increasing in q, while the right hand side is constant in q. It is obvious that a strictly increasing function intersects with a constant function only once, which means that there is a unique q that satisfies equation 6.11. That unique value of q corresponds to a unique value of p.

Second, we show that value of Q_0 that solves equations 6.9 and 6.10 is unique. First, prove that the payoff in equation 6.9 is decreasing in Q_0.[†] Namely, from Figure 6.2 one can see that when we increase Q_0 we move curve Q_0Q_{n-1} higher, and this change can only decrease the payoff given both curves are increasing.[†] Second, in equation 6.10 the left hand side is decreasing in Q_0, while the right hand side is increasing in Q_0. There is a unique value of Q_0 that satisfies this equation. Finally, let us prove that both curves in Figure 6.2 are increasing. Coefficients for the left and right hand sides of equation 6.9 are positive and increasing. It is a simple exercise to take derivatives from these functions with respect to p and show that the derivatives are always positive. This fact concludes the proof. □

Result 1 *In the original set-up once the game switches from a prisoners' dilemma to a coordination game, the players optimal strategies will remain a coordination game in every subsequent period as the number of periods is increased.*

Proof. Let us firstly prove the proposition when there are only two players. If players face a prisoners' dilemma in period $n-1$, while in period n they face a coordination game then

$$R_1 = Q_0 * d^{-\alpha} = R_2 * d^{n-\alpha}, \ \ \alpha \in (0,1). \tag{6.12}$$

where R_1 is the payoff of a player if he invests in the first period and the other player does not invest; R_2 is the payoff of a player when both players invest; Q_0 is

[†]Note, that from the previous paragraph it follows that the payoff is determined uniquely for any given value of Q_0.

[†]In Figure 6.2 we depict the left and right hand sides of equation 6.9 as functions of p. The values of these functions at the point where they intersect are equal to the payoff.

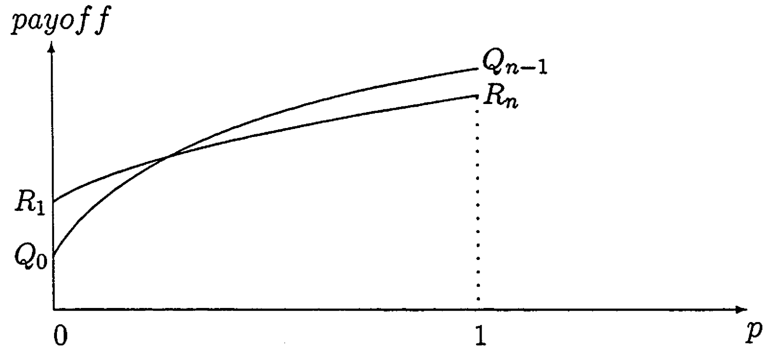

Figure 6.2: Graphical interpretation why the payoff is decreasing in Q_0

the payoff of a player if both do not invest; and finally Q_1 is the payoff of a player if he does not invest while the other player invests.

If in period $n+1$ the game will again be a prisoners' dilemma, then the mixed strategy return is greater than R_1. This requires that

$$\frac{R_1Q_1 - R_2Q_0}{R_1 + Q_1 - R_2 - Q_0} d > R_1. \tag{6.13}$$

Using the fact that

$$Q_0 = d^n R_2, \tag{6.14}$$

inequality 6.13 can be written as

$$(Q_1 - R_2 * d^{\alpha})d > Q_1 - R_2 + (d^{-\alpha} - 1)d^n R_2. \tag{6.15}$$

Now, let us assume that

$$R_1 + Q_1 = 2R_2. \tag{6.16}$$

This allows inequality 6.15 to be written as

$$(2 - d^{\alpha})d - 1 > d^n(d^{1-\alpha} - 1). \tag{6.17}$$

When $n = 0$, inequality 6.17 transforms to

$$2d > d^{1+\alpha} + d^{1-\alpha} \tag{6.18}$$

which contradicts the fact that the geometrical average is always less than arithmetical one. To get a similar contradiction for $n > 0$, notice that the right hand side in equation 6.17 is negative and increasing with n.

Thus, when $R_1 + Q_1 = 2R_2$ there is no immediate jump to the prisoners' dilemma when there are only two players and the following inequality is satisfied

$$(Q_1 - R_2 * d^{\alpha})d < Q_1 - R_2 + (d^{-\alpha} - 1)d^n R_2. \tag{6.19}$$

This inequality is opposite to inequality 6.15 and follows from the above contradiction. When $R_1 + Q_1 > 2R_2$ the same inequality holds because the derivative of inequality 6.19 with respect to Q_1 is negative.

Now the question is whether we can jump in later periods. The answer will be 'no' if the mixed return is greater than $R_1 d$, which follows straight away from inequality 6.13. Namely, we showed that

$$R_1 > \frac{R_1 Q_1 - R_2 Q_0}{R_1 + Q_1 - R_2 - Q_0} d > R_1 d, \tag{6.20}$$

which means that the mixed return can be written as $R_1 d^{-\alpha_2}$. This mixed return is actually Q_0 for the next period. Thus, the same logic works for the non-immediate jump as well.

Next we prove that the same result holds with more than 2 players.

Let R_k denote the payoff of a player if he invests and Q_k denote the payoff if he does not invest; k indicates the number of players investing. We assume that

$$Q_{n-1} \geq \dots Q_1 \geq R_n \geq \dots \geq R_1 \geq Q_0. \tag{6.21}$$

Let us find the mixed strategy equilibrium. The probability of investing solves the following equation

$$\begin{aligned}(1-p)^{n-1}R_1 + Q_{n-1}^1 p(1-p)^{n-2}R_2 + \ldots + p^{n-1}R_n &= \\ (1-p)^{n-1}Q_0 + Q_{n-1}^1 p(1-p)^{n-2}Q_1 + \ldots + p^{n-1}Q_{n-1}.\end{aligned} \tag{6.22}$$

The payoff is either the left hand side or right hand side of this equation. To show when $d * payoff < R_1$, let us show when this payoff is less than the payoff from two player game with Q_{n-1}, Q_0, R_n and R_1. For the moment assume that

$$R_2 - Q_1 = R_3 - Q_2 = \ldots = R_n - Q_{n-1}. \tag{6.23}$$

In this case it is easy to see that the payoff has a maximum when $Q_1 = Q_2 = \ldots = Q_{n-1}$ and $R_2 = R_3 = \ldots = R_n$, and this maximum is equal to the payoff from the two player game. Now equation 6.22 can be written as

$$\frac{(1-p)^{n-1}}{1-(1-p)^{n-1}} = \frac{R_n - Q_{n-1}}{R_1 - Q_0}. \tag{6.24}$$

Using equation 6.22 one can calculate the payoff, and it will be exactly the same as the payoff from two player game with Q_{n-1}, Q_0, R_n and R_1. Thus, when condition 6.23 holds, there is no return to the prisoner's dilemma.

Now let consider a general case when condition 6.23 does not hold. Here we use a similar technique that we used when we proved Proposition 6.2. The left hand side and the right hand side of equation 6.22 are increasing functions with respect to p. From Figure 6.2 one can see that when we decrease R_i $i = 2 \ldots n-1$ we move curvè R_1R_n lower and this change decreases the payoff. It means that for values

$$R_2 - Q_1 \leq R_n - Q_{n-1}, \;\; R_3 - Q_2 \leq R_n - Q_{n-1} \ldots \tag{6.25}$$

there is no return to the prisoner's dilemma. This observation concludes the proof. □

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